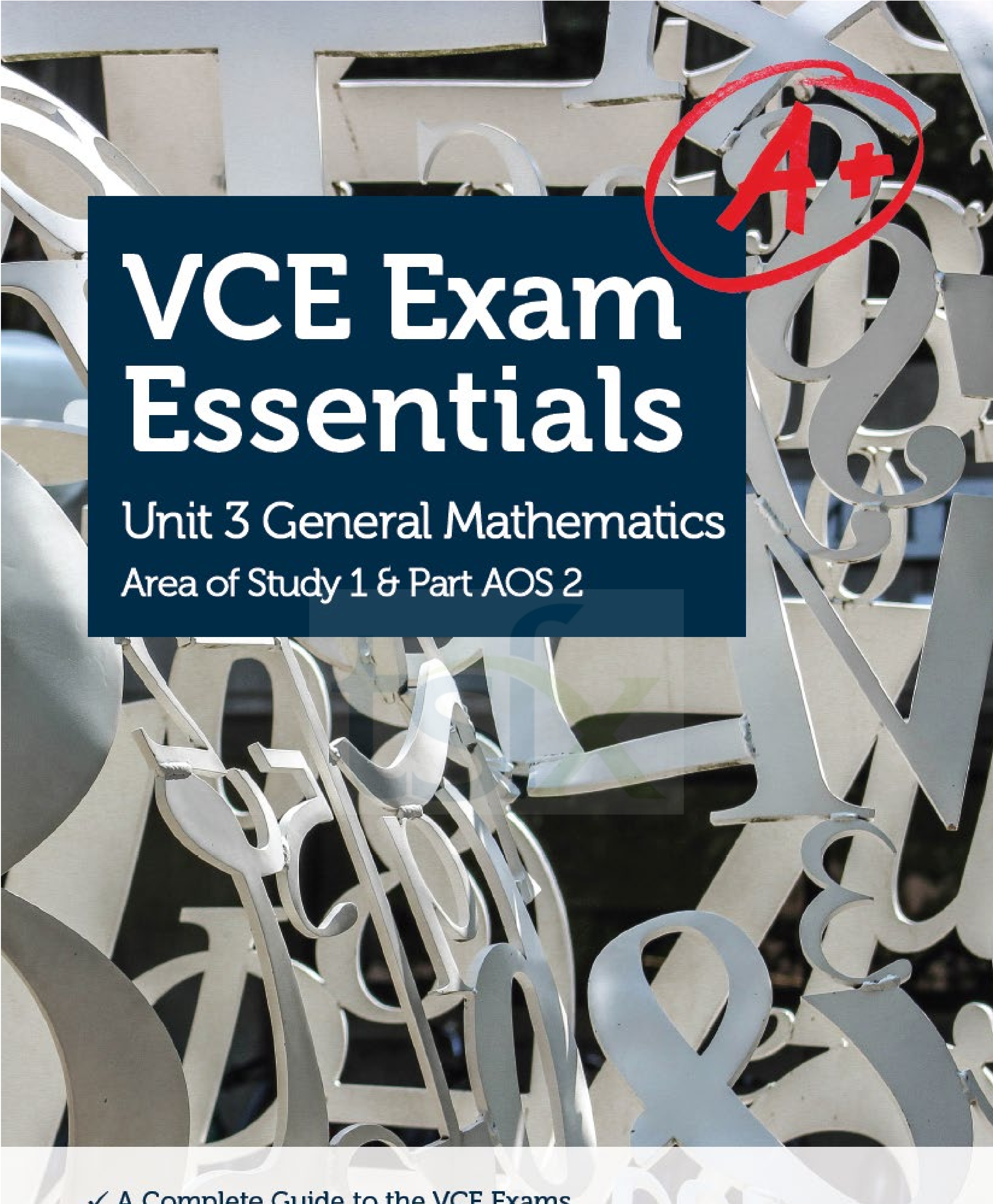


VCE Exam Essentials

Unit 3 General Mathematics

Area of Study 1 & Part AOS 2

- ✓ A Complete Guide to the VCE Exams
- ✓ Comprehensive & Detailed A+ Notes
- ✓ VCE Exam Style Questions & Solutions
- ✓ Written by Experienced VCAA Examiners
- ✓ VCAA Examination Standard

VCE EXAM ESSENTIALS

Unit 3 General Mathematics

VCE Accreditation Period
2023 – 2027

www.tsfx.edu.au/vce-essentials

TSFX

Suite 101 & 102
964 Mt Alexander Rd
Essendon, VIC 3040

Phone: (03) 9663 3311
Email: admin@tsfx.edu.au
Web: www.tsfx.edu.au

ABN: 70 066 955 128

www.tsfx.edu.au

Acknowledgements

The publisher wishes to thank the Victorian Curriculum and Assessment Authority for permission to reproduce extracts from their publications.

Errata & Bonus Resources

Although great care is taken to ensure that our books are mistake free, an error may appear from time to time. If you believe there is an error in this publication, please let us know asap **(admin@tsfx.edu.au)**.

Errors, as well as additional resources that become available can be found at:

www.tsfx.edu.au/vce-essentials.

CONTENTS

UNIT 3: GENERAL MATHEMATICS

PART 1: VCE STUDY DESIGN

Unit 3 & Unit 4: General Mathematics VCE Study Design **Page 2**

PART 2: AREA OF STUDY 1
DATA ANALYSIS, PROBABILITY & STATISTICS

Section 1: Investigating Data Distributions **Page 19**

Types of Data Page 19
Types of Variables Page 19

Univariate Analysis **Page 23**

Organising Data Page 23
Stem Plots (Stem & Leaf Plots) Page 25
Bar Graphs Page 25
Segmented Bar Charts Page 26
Dot Plots Page 26
Histograms Page 27
Using Logarithmic Scales Page 35
Shapes of Distribution Page 40

Measures of Central Tendency **Page 44**

Mode Page 44
Median Page 44
Mean Page 45

Measures of Spread **Page 54**

The Range Page 54
Interquartile Range Page 54
Boxplots Page 58
The Standard Deviation Page 64
The Normal Model for a Bell-Shaped Distribution Page 66
Standard (Z) Scores Page 70

Section 2: Investigating Association Between Two Variables **Page 77**

Bivariate Data **Page 77**

Bivariate Data – Two Categorical Variables Page 77
Bivariate Data – One Categorical and One Numerical Variable Page 80
Bivariate Analysis – Numerical and Numerical Page 84
Scatterplots Page 86
The Strength of a Linear Relationship Page 89
Person's Product Moment Correlation Coefficient of the r Value Page 89
The Coefficient of Determination Page 91
Correlation & Causation Page 95

Section 3: Investigating & Modelling Linear Associations **Page 96**

Regression – Least-Square Regression Line Page 96
Calculating the Least-Squares from Summary Data Page 96
Calculating the Least-Squares Line from Raw Data Page 100
Interpolation, Extrapolation & Interpretation Page 104
Residuals and Residual Plots Page 106
Types of Residual Plots Page 107

Transformations **Page 112**

Transformations Using y^2 or x^2 Page 114
Transformations Using $\frac{1}{y}$ or $\frac{1}{x}$ Page 115
Transformations Using $log_{10}y$ or $log_{10}x$ Page 116
Choosing the Correct Transformations Page 118
Testing Transformations Page 121

Section 4: Investigating & Modelling Time Series Data **Page 127**

Displaying, Summarising & Describing Times Series Data Page 127
Trends Page 127
Cycles Page 128
Seasonality Page 128
Structural Changes Page 129
Outliers Page 129
Irregular (Random) Fluctuations Page 130

Smoothing **Page 131**

Median Smoothing Page 132
Moving Mean Smoothing Page 134

Deseasonalising Data **Page 139**

Predicting from Time Series **Page 141**

PART 3: AREA OF STUDY 2
RECURSION & FINANCIAL MODELLING

Section 1: Deprecation of Assets **Page 152**

Recursion Page 152
Investment & Loans Page 152
Recursion Page 153
Investments Page 155

Simple Interest Page 155

Section 2: Compound Interest Investments & Loans **Page 161**

Compound Interest Page 161
Nominal & Effective Interest Rates Page 167

Depreciation of Assets **Page 170**

Flat Rate Depreciation Page 170
Reducing Balance Depreciation Page 175
Unit Cost Depreciation Page 180

Section 3: Reducing Balance Loans **Page 183**

The Amortisation of a Loan Page 187
Interest Only Loans Page 195

Section 4: Compound Interest Investment with Periodic & Equal Additions to the Principal **Page 198**

Annuity Investments Page 198

Section 5: Annuity & Perpetuities **Page 202**

Annuities Page 202
Perpetuities Page 204

Section 6: Additional Questions **Page 207**

PART 4: SOLUTIONS

Part 2: Data Analysis Page 214
Part 3: Recursion & Financial Modelling Page 233
Additional Questions Page 242

PART 1: VCE STUDY DESIGN

UNIT 3 & UNIT 4
GENERAL MATHEMATICS

ACCREDITATION PERIOD
2023 – 2027

UNITS 3 & UNIT 4: GENERAL MATHEMATICS
VCE STUDY DESIGN

ACCREDITATION PERIOD: 2023 – 2027

General Mathematics Units 3 and 4 focus on real-life application of mathematics and consist of the areas of study 'Data analysis, probability and statistics' and 'Discrete mathematics'. Unit 3 comprises *Data analysis* and *Recursion and financial modelling*, and Unit 4 comprises *Matrices* and *Networks and decision mathematics*.

Assumed knowledge and skills for General Mathematics Units 3 and 4 are contained in General Mathematics Units 1 and 2, and will be drawn on, as applicable, in the development of related content from the areas of study, and key knowledge and key skills for the outcomes of General Mathematics Units 3 and 4.

In undertaking these units, students are expected to be able to apply techniques, routines and processes involving rational and real arithmetic, sets, lists, tables and matrices, diagrams, networks, algorithms, algebraic manipulation, recurrence relations, equations and graphs. They should have facility with relevant mental and by-hand approaches to estimation and computation. The use of numerical, graphical, geometric, symbolic statistical and financial functionality of technology for teaching and learning mathematics, for working mathematically, and in related assessment, is to be incorporated throughout each unit as applicable.

AREA OF STUDY 1
DATA ANALYSIS, PROBABILITY & STATISTICS

DATA ANALYSIS

Students cover data types, representation and distribution of data, location, spread, association, correlation and causation, response and explanatory variables, linear regression, data transformation and goodness of fit, times series, seasonality, smoothing and prediction.

INVESTIGATING DATA DISTRIBUTIONS

This topic includes:

- Types of data.
- Representation, display and description of the distributions of categorical variables: data tables, two-way frequency tables and their associated segmented bar charts.
- Representation, display and description of the distributions of numerical variables: dot plots, stem plots, histograms; the use of a logarithmic (base 10) scale to display data ranging over several orders of magnitude and their interpretation in terms of powers of ten.
- Use of the distribution(s) of one or more categorical or numerical variables to answer statistical questions.
- Summary of the distributions of numerical variables; the five-number summary and boxplots (including the use of the lower fence ($Q1 - 1.5 \times IQR$) and upper fence ($Q3 + 1.5 \times IQR$) to identify and display possible outliers); the sample mean and standard deviation and their use in comparing data distributions in terms of centre and spread.

- The normal model for bell-shaped distributions and the use of the 68–95–99.7% rule to estimate percentages and to give meaning to the standard deviation; standardised values (z-scores) and their use in comparing data values across distributions.

INVESTIGATING ASSOCIATION BETWEEN TWO VARIABLES

This topic includes:

- Response and explanatory variables and their role in investigating associations between variables.
- Contingency (two-way) frequency tables, their associated bar charts (including percentage segmented bar charts) and their use in identifying and describing associations between two categorical variables.
- Back-to-back stem plots, parallel dot plots and boxplots and their use in identifying and describing associations between a numerical variable and a categorical variable.
- Scatterplots and their use in identifying and qualitatively describing the association between two numerical variables in terms of direction (positive/negative), form (linear/non-linear) and strength (strong/moderate/weak).
- Answering statistical questions that require a knowledge of the associations between pairs of variables.
- Pearson correlation coefficient, r, and its calculation and interpretation.
- Cause and effect; the difference between observation and experimentation when collecting data and the need for experimentation to definitively determine cause and effect.

INVESTIGATING & MODELLING LINEAR ASSOCIATIONS

This topic includes:

- Least squares line of best fit $y = a + bx$, where x represents the explanatory variable, and y represents the response variable; the determination of the coefficients a and b using technology, and the formulas $b = r\,\frac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$.
- Modelling linear association between two numerical variables, including the:

  Identification of the explanatory and response variables.

  Use of the least squares method to fit a linear model to the data.
- Interpretation of the slope and intercepts of the least squares line in the context of the situation being modelled, including:

  Use of the rule of the fitted line to make predictions being aware of the limitations of extrapolation.

  Use of the coefficient of determination, r^2, to assess the strength of the association in terms of explained variation.

  Use of residual analysis to check quality of fit.

- Data transformation and its use in transforming some forms of non-linear data to linearity using a square, logarithmic (base 10) or reciprocal transformation (applied to one axis only).

- Interpretation and use of the equation of the least squares line fitted to the transformed data to make predictions.

INVESTIGATING & MODELLING TIME SERIES DATA

This topic includes:

- Qualitative features of time series plots; recognition of features such as trend (long-term direction), seasonality (systematic, calendar related movements) and irregular fluctuations (unsystematic, short-term fluctuations); possible outliers and their sources, including one-off real-world events, and signs of structural change such as a discontinuity in the time series.

- Numerical smoothing of time series data using moving means with consideration of the number of terms required (using centring when appropriate) to help identify trends in time series plot with large fluctuations.

- Graphical smoothing of time series plots using moving medians (involving an odd number of points only) to help identify long-term trends in time series with large fluctuations.

- Seasonal adjustment including the use and interpretation of seasonal indices and their calculation using seasonal and yearly means.

- Modelling trend by fitting a least squares line to a time series with time as the explanatory variable (data de-seasonalised where necessary), and the use of the model to make forecasts (with re-seasonalisation where necessary) including consideration of the possible limitations of fitting a linear model and the limitations of extending into the future.

AREA OF STUDY 2
DISCRETE MATHEMATICS

RECURSION & FINANCIAL MODELLING

Students cover the use of first-order linear recurrence relations and the time value of money (TVM) to model and analyse a range of financial situations, and using technology to solve related problems involving interest, appreciation and depreciation, loans, annuities and perpetuities.

DEPRECIATION OF ASSETS

This topic includes:

- Use of a first-order linear recurrence relation of the form: $u_0 = a, u_{n+1} = Ru_n + d$ where a, R and d are constants to generate the terms of a sequence.
- Use of a recurrence relation to model and compare (numerically and graphically) flat rate, unit cost and reducing balance depreciation of the value of an asset with time, including the use of a recurrence relation to determine the depreciating value of an asset after n depreciation periods for the initial sequence.
- Use of the rules for the future value of an asset after n depreciation periods for flat rate, unit cost and reducing balance depreciation and their application.

COMPOUND INTEREST INVESTMENTS & LOANS

This topic includes:

- The concepts of simple and compound interest.
- Use of a recurrence relation to model and analyse (numerically and graphically) a compound interest investment or loan, including the use of a recurrence relation to determine the value of the compound interest loan or investment after n compounding period for an initial sequence from first principles.
- The difference between nominal and effective interest rates and the use of effective interest rates to compare investment returns and the cost of loans when interest is paid or charged, for example, daily, monthly, quarterly.
- The future value of a compound interest investment or loan after n compounding periods and its use to solve practical problems.

REDUCING BALANCE LOANS

This topic includes:

- Use of a first-order linear recurrence relation to model and analyse (numerically and graphically) the amortisation of a reducing balance loan, including the use of a recurrence relation to determine the value of the loan or investment after n payments for an initial sequence from first principles.
- Use of a table to investigate and analyse the amortisation of a reducing balance loan on a step-by-step basis, the payment made, the amount of interest paid, the reduction in the principal and the balance of the loan.

- Use of technology with financial modelling functionality to solve problems involving reducing balance loans, such as repaying a personal loan or a mortgage, including the impact of a change in interest rate on repayment amount, time to repay the loan, total interest paid and the total cost of the loan.

ANNUITIES & PERPETUITIES

This topic includes:

- Use of a first-order linear recurrence relation to model and analyse (numerically and graphically) the amortisation of an annuity, including the use of a recurrence relation to determine the value of the annuity after n payments for an initial sequence from first principles.

- Use of a table to investigate and analyse the amortisation of an annuity on a step-by-step basis, the payment made, the interest earned, the reduction in the principal and the balance of the annuity.

- Use of technology to solve problems involving annuities including determining the amount to be invested in an annuity to provide a regular income paid, for example, monthly, quarterly.

- Simple perpetuity as a special case of an annuity that lasts indefinitely.

COMPOUND INTEREST INVESTMENT WITH PERIODIC & EQUAL ADDITIONS TO THE PRINCIPAL

This topic includes:

- Use of a first-order linear recurrence relation to model and analyse (numerically and graphically) annuity investment, including the use of a recurrence relation to determine the value of the investment after n payments have been made for an initial sequence from first principles.

- Use of a table to investigate and analyse the growth of an annuity investment on a step-by-step basis after each payment is made, the payment made, the interest earned and the balance of the investment.

- Use of technology with financial modelling functionality to solve problems involving annuity investments, including determining the future value of an investment after a number of compounding periods, the number of compounding periods for the investment to exceed a given value and the interest rate or payment amount needed for an investment to exceed a given value in a given time.

MATRICES

Students cover the definition of matrices, different types of matrices, matrix operations, transition matrices and the use of first-order linear matrix recurrence relations to model a range of situations and solve related problems.

MATRICES & THEIR APPLICATIONS

This topic includes:

- Matrix arithmetic: the order of a matrix, types of matrices (row, column, square, diagonal, symmetric, triangular, zero, binary and identity), the transpose of a matrix, and elementary matrix operations (sum, difference, multiplication of a scalar, product and power).
- Inverse of a matrix, its determinant, and the condition for a matrix to have an inverse.
- Use of matrices to represent numerical information presented in tabular form, and the use of a rule for the $a_{ij}\text{th}$ element of a matrix to construct the matrix.
- Binary and permutation matrices, and their properties and applications.
- Communication and dominance matrices and their use in analysing communication systems and ranking players in round-robin tournaments.

TRANSITION MATRICES

This topic includes:

- Use of the matrix recurrence relation: $S_0 =$ initial state matrix, $S_{n+1} = TS_n$ or $S_{n+1} = LS_n$ where T is a transition matrix, L is a leslie matrix, and S_n is a column state matrix, to generate a sequence of state matrices (assuming the next state only relies on the current state).
- Informal identification of the equilibrium state matrix in the case of regular transition matrices (no noticeable change from one state matrix to the next state matrix).
- Use of transition diagrams, their associated transition matrices and state matrices to model the transitions between states in discrete dynamical situations and their application to model and analyse practical situations such as the modelling and analysis of an insect population comprising eggs, juveniles and adults.
- Use of the matrix recurrence relation $S_0 =$ initial state matrix, $S_{n+1} = TS_n + B$ to extend modelling to populations that include culling and restocking.

NETWORKS & DECISION MATHEMATICS

Students cover the definition and representation of different kinds of undirected and directed graphs, Eulerian trails, Eulerian circuits, bridges, Hamiltonian paths and cycles, and the use of networks to model and solve problems involving travel, connection, flow, matching, allocation and scheduling.

GRAPHS & NETWORKS

This topic includes:

- The concepts, conventions and terminology of graphs including planar graphs and Euler's rule and directed (digraphs) and networks.
- Use of matrices to represent graphs, digraphs and networks and their application.

EXPLORING & TRAVELLING PROBLEMS

This topic includes:

- The concepts, conventions and notations of walks, trails, paths, cycles and circuits.
- Eulerian trails and Eulerian circuits: the conditions for a graph to have a Eulerian trail or a Eulerian circuit, properties, and applications.
- Hamiltonian paths and cycles: properties and applications.

TREES & MINIMUM CONNECTOR PROBLEMS

This topic includes:

- Trees and spanning trees.
- Minimum spanning trees in a weighed connected graph and their determination by inspection or by prim's algorithm.
- Use of minimal spanning trees to solve minimal connector problems.

FLOW PROBLEMS

This topic includes:

- Use of networks to model flow problems: capacity, sinks and sources.
- Solution of small-scale network flow problems by inspection and the use of the 'maximum-flow minimum-cut' theorem to aid the solution of larger scale problems.

SHORTEST PATH PROBLEMS

This topic includes:

- Determination of the shortest path between two specified vertices in a graph, digraph or network by inspection.
- Dijkstra's algorithm and its use to determine the shortest path between a given vertex and each of the other vertices in a weighted graph or network.

MATCHING PROBLEMS

This topic includes:

- Use of a bipartite graph and its tabular or matrix form to represent a matching problem.
- Determination of the optimum assignment(s) of people or machines to tasks by inspection or by use of the Hungarian algorithm for larger scale problems.

SCHEDULING PROBLEMS & CRITICAL PATH ANALYSIS

This topic includes:

- Construction of an activity network from a precedence table (or equivalent) including the use of dummy activities where necessary.
- Use of forward and backward scanning to determine the earliest starting times (EST) and latest starting times (LST) for each activity.
- Use of earliest starting times and latest starting times to identify the critical path in the network and determine the float times for non-critical activities.
- Use of crashing to reduce the completion time of the project or task being modelled.

OUTCOMES

For each unit the student is required to demonstrate achievement of three outcomes.
As a set these outcomes encompass all of the areas of study for each unit. For each of Unit 3 and Unit 4 the outcomes as a set apply to the content from the areas of study covered in that unit.

OUTCOME 1

On completion of this unit the student should be able to define and explain key concepts as specified in the content from the areas of study and apply a range of related mathematical routines and procedures.

To achieve this outcome the student will draw on key knowledge and key skills outlined in all the areas of study.

AREA OF STUDY 1 – DATA ANALYSIS, PROBABILITY & STATISTICS

KEY KNOWLEDGE: OUTCOME 1

- Types of data: categorical (nominal and ordinal) and numerical (discrete and continuous).
- Frequency tables, bar charts including segmented bar charts, histograms, stem plots, dot plots, and their application in the context of displaying and describing distributions.
- Logarithmic (base 10) scales, and their purpose and application.
- The five-number summary and boxplots (including the designation and display of possible outliers).
- Mean $\bar{x}$ and sample standard deviation $s = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}}$.
- The normal model and the 68–95–99.7% rule, and standardised values (z-scores).
- Response and explanatory variables.
- Two-way frequency tables, segmented bar charts, back-to-back stem plots, parallel boxplots, and scatterplots, and their application in the context of identifying and describing associations.
- Correlation coefficient, r, its interpretation, the issue of correlation and cause and effect.
- Coefficient of determination, its interpretation.
- Least squares line and its use in modelling linear associations.
- Data transformation and its purpose.
- Time series data and its analysis.

KEY SKILLS: OUTCOME 1

- Construct frequency tables and bar charts and use them to describe and interpret the distributions of categorical variables.
- Answer statistical questions that require a knowledge of the distribution(s) of one or more categorical variables.
- Construct stem and dot plots, boxplots, histograms and appropriate summary statistics and use them to describe and interpret the distributions of numerical variables.
- Answer statistical questions that require a knowledge of the distribution(s) of one or more numerical variables.
- Solve problems using z-scores and the 68–95–99.7% rule.

- Construct two-way tables and use them to identify and describe associations between two categorical variables.

- Construct parallel boxplots and use them to identify and describe associations between a numerical variable and a categorical variable.

- Construct scatterplots and use them to identify and describe associations between two numerical variables.

- Calculate the correlation coefficient, r, and interpret it in the context of the data.

- Answer statistical questions that require a knowledge of the associations between pairs of variables.

- Determine the equation of the least squares line giving the coefficients correct to a required number of decimal places or significant figures as specified and distinguish between correlation and causation.

- Use the least squares line of best fit to model and analyse the linear association between two numerical variables and interpret the model in the context of the association being modelled.

- Calculate the coefficient of determination, r^2 , and interpret in the context of the association being modelled and use the model to make predictions, being aware of the problem of extrapolation.

- Construct a residual analysis to test the assumption of linearity and, in the case of clear non-linearity, transform the data to achieve linearity and repeat the modelling process using the transformed data.

- Identify key qualitative features of a time series plot including trend (using smoothing if necessary), seasonality, irregular fluctuations and outliers, and interpret these in the context of the data.

- Calculate, interpret and apply seasonal indices.

- Model linear trends using the least squares line of best fit, interpret the model in the context of the trend being modelled, use the model to make forecasts with consideration of the limitations of extending forecasts too far into the future.

AREA OF STUDY 2 – DISCRETE MATHEMATICS

RECURSION & FINANCIAL MODELLING

KEY KNOWLEDGE: OUTCOME 1

- The use of first-order linear recurrence relations to model growth and decay problems in financial contexts.
- The use of first-order linear recurrence relations to model flat rate and unit cost, and reduce balance depreciation of an asset over time, including the rule for the future value of the asset after n depreciation periods.
- The concepts of financial mathematics including simple and compound interest, nominal and effective interest rates, the present and future value of an investment, loan or asset, amortisation of a reducing balance loan or annuity and amortisation tables.
- The use of first-order linear recurrence relations to model compound interest investments and loans, and the flat rate, unit cost and reducing balance methods for depreciating assets, reducing balance loans, annuities, perpetuities, and annuity investments.

KEY SKILLS: OUTCOME 1

- Model and analyse growth and decay in financial contexts using a first-order linear recurrence relation of the form: $u_0 = a$, $u_{n+1} = Ru_n + d$ where a, R and d are constants.
- Demonstrate the use of a recurrence relation to determine the depreciating value of an asset or the future value of an investment or a loan after n time periods for the initial sequence.
- Use a rule for the future value of a compound interest investment or loan, or a depreciating asset, to solve practical problems.
- Use a table to investigate and analyse on a step–by-step basis the amortisation of a reducing balance loan or an annuity and interpret amortisation tables.
- Use technology with financial mathematics capabilities, to solve practical problems associated with compound interest investments and loans, reducing balance loans, annuities and perpetuities, and annuity investments.

MATRICES

KEY KNOWLEDGE: OUTCOME 1

- The order of a matrix, types of matrices (row, column, square, diagonal, symmetric, triangular, zero, binary, permutation and identity), the transpose of a matrix, and elementary matrix operations (sum, difference, multiplication of a scalar, product and power).
- The inverse of a matrix and the condition for a matrix to have an inverse, including determinant.
- For transition matrices, assuming the next state only relies on the current state with a fixed population.
- Communication and dominance matrices and their application.
- Transition diagrams and transition matrices and regular transition matrices and their identification.

KEY SKILLS: OUTCOME 1

- Use matrix recurrence relations to generate a sequence of state matrices, including an informal identification of the equilibrium or steady state matrix in the case of regular state matrices.
- Construct a transition matrix from a transition diagram or a written description and vice versa.
- Construct a transition matrix to model the transitions in a population with an equilibrium state.
- Use matrix recurrence relations to model populations with culling and restocking.

NETWORKS AND DECISION MATHEMATICS

KEY KNOWLEDGE: OUTCOME 1

- The conventions, terminology, properties and types of graphs; edge, face, loop, vertex, the degree of a vertex, isomorphic and connected graphs, and the adjacency matrix, and Euler's formula for planar graphs and its application.
- The exploring and travelling problem, walks, trails, paths, Eulerian trails and circuits, and Hamiltonian paths and cycles.
- The minimum connector problem, trees, spanning trees and minimum spanning trees and Prim's algorithm.
- The flow problem, and the minimum cut/maximum flow theorem.
- The shortest path problem and Dijkstra's algorithm.
- The matching problem and the Hungarian algorithm.
- The scheduling problem and critical path analysis.

KEY SKILLS: OUTCOME 1

- Construct graphs, digraphs and networks and their matrix equivalents to model and analyse practical situations.
- Recognise the exploring and travelling problem and to solve it by utilising the concepts of walks, trails, paths, Eulerian trails and circuits, and Hamiltonian paths and cycles.
- Recognise the minimum connector problem and solve it by utilising the properties of trees, spanning trees and by determining a minimum spanning tree by inspection or using prim's algorithm for larger scale problems.
- Recognise the flow problem, use networks to model flow problems and determine the minimum flow problem by inspection, or by using the minimum cut/maximum flow theorem for larger scale problems.
- Recognise the shortest path problem and solve it by inspection or using Dijkstra's algorithm for larger scale problems.
- Recognise the matching problem and solve it by inspection or using the Hungarian algorithm for larger scale problems.
- Recognise the scheduling problem and solve it by using critical path analysis.

OUTCOME 2

On completion of this unit the student should be able to apply mathematical processes in non-routine contexts, including situations with some open-ended aspects requiring investigative, modelling or problem-solving techniques or approaches, and analyse and discuss these applications of mathematics.

To achieve this outcome the student will draw on key knowledge and key skills outlined in all the areas of study.

KEY KNOWLEDGE: OUTCOME 2

- The facts, concepts and techniques associated with data analysis, recursion and financial modelling, matrices and networks and decision mathematics.
- Standard models studied in data analysis, recursion and financial modelling, matrices, and networks and decision mathematics, and their areas of application.
- General formulation of the concepts, techniques and models studied in data analysis, recursion and financial modelling, matrices, and networks and decision mathematics.
- Assumptions and conditions underlying the use of the concepts, techniques and models associated with data analysis, recursion and financial modelling, matrices, and networks and decision mathematics.

KEY SKILLS: OUTCOME 2

- Identify, recall and select facts, concepts, models and techniques needed to investigate and analyse statistical features of a data set with several variables that can include time series data.
- Select and implement standard financial models to investigate and analyse a financial or mathematically equivalent non-financial situation that requires the use of increasingly sophisticated models to complete the analysis.
- Identify, recall and select the mathematical concepts, models and techniques needed to solve an extended problem or conduct an investigation in a variety of contexts related to matrices and networks and decision mathematics.
- Interpret and report the results of a statistical investigation or of completing a modelling or problem-solving task in terms of the context under consideration, including discussing the assumptions in application of these models.

OUTCOME 3

On completion of this unit the student should be able to apply computational thinking and use numerical, graphical, symbolic and statistical functionalities of technology to develop mathematical ideas, produce results and carry out analysis in situations requiring investigative, modelling or problem-solving techniques or approaches.

To achieve this outcome the student will draw on key knowledge and key skills outlined in all the areas of study.

KEY KNOWLEDGE: OUTCOME 3

- The role of computational thinking (abstraction, decomposition, pattern and algorithm) in problem-solving, and its application to mathematical investigation.
- The difference between exact numerical and approximate numerical answers when using technology to perform computation and rounding to a given number of decimal places or significant figures.
- Domain and range requirements for specification of graphs of models and relations, when using technology.
- The role of parameters in specifying general forms of models and equations.
- The relation between numerical, graphical and symbolic forms of information about models and equations, and the corresponding features of those models and equations.
- The similarities and differences between formal mathematical expressions and their representation by technology.
- The appropriate functionality of technology for a variety of mathematical contexts.

KEY SKILLS: OUTCOME 3

- Use computational thinking, algorithms, models and simulations to solve problems related to a given context.
- Distinguish between exact and approximate presentations of mathematical results produced by technology and interpret these results to a specified degree of accuracy in terms of a given number of decimal places or significant figures.
- Use technology to carry out numerical, graphical and symbolic computation as applicable.
- Produce results, using a technology, which identify examples or counter-examples for propositions.
- Produce tables of values, families of graphs and collections of other results using technology, which support general analysis in investigative, modelling and problem-solving contexts.
- Use appropriate domain and range specifications to illustrate key features of graphs.

- Identify the relation between numerical, graphical and symbolic forms of information about models and equations, and the corresponding features of those models and equations.
- Specify the similarities and differences between formal mathematical expressions and their representation by technology.
- Select an appropriate functionality of technology in a variety of mathematical contexts, related to data analysis, recurrence relations and financial modelling, and provide a rationale for these selections.
- Apply suitable constraints and conditions, as applicable, to carry out required computations.
- Relate the results from a particular technology application to the nature of a particular mathematical task (investigative, modelling or problem-solving) and verify these results.
- Specify the process used to develop a solution to a problem using technology and communicate the key stages of mathematical reasoning (formulation, solution, interpretation) used in this process.

PART 2: AREA OF STUDY 1

DATA ANALYSIS, PROBABILITY & STATISTICS

DATA ANALYSIS

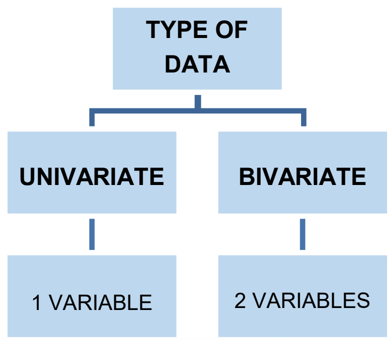
PART 2: AREA OF STUDY 1
DATA ANALYSIS

SECTION 1: INVESTIGATING DATA DISTRIBUTIONS

TYPES OF DATA

Univariate data – Examines the distribution features of one variable.

Bivariate data – Explores the relationship between two variables.



Univariate and bivariate analysis will be revised separately.

TYPES OF VARIABLES

NUMERICAL VARIABLES

- **Numerical variables** represent quantities. They have numerical values. They are measured or counted.
- Numerical variables can either be **continuous** or **discrete**.
- A **continuous variable** can take any value in a given range. They are usually **measured**.
- Examples: Height, weight, number of litres of fuel.
- A **discrete variable** can take on only certain distinct values in a given range. They are often **counted**. i.e. 0, 1, 2, 3, etc.
- **Examples:** Number of siblings, number of goals in netball.

CATEGORICAL VARIABLES

- **Categorical variables** represent qualities. The answer to the statistical question is usually a word rather than a number. They can be nominal or ordinal.

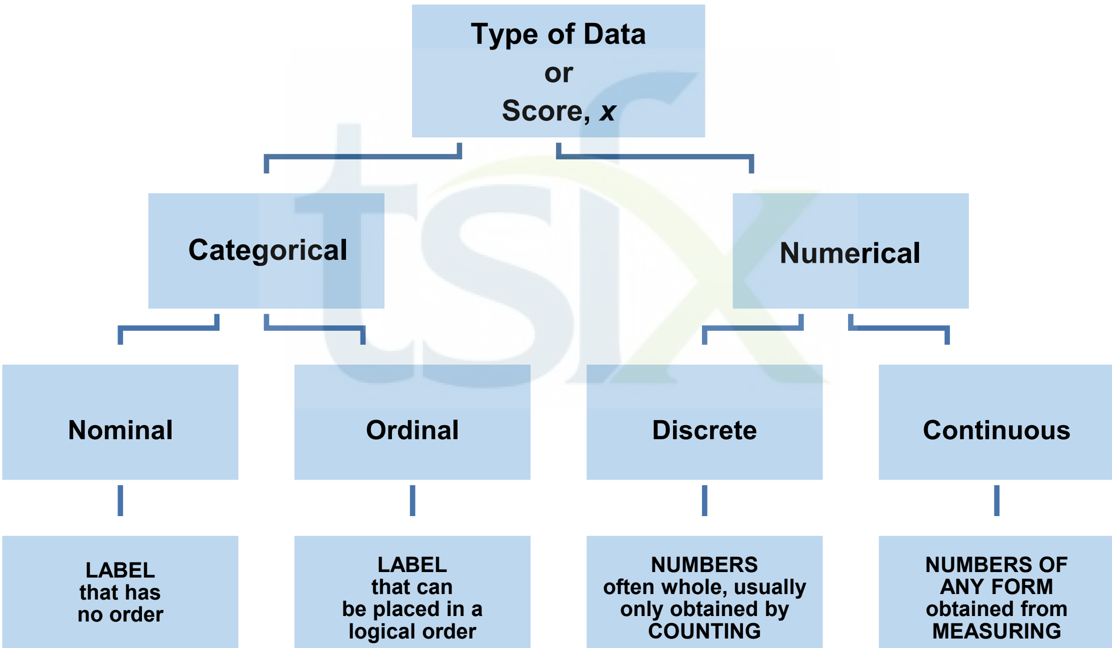
- **Ordinal categorical variables** have a natural order such as salary (high, medium or low). Sometimes numerical values or scores can be assigned to ordinal data, for example such as 'tidies up his/her room' (1 = regularly, 2 = sometimes, 3 = rarely, 4 = never). However, these numerical values are artificial and therefore such variables are still considered categorical.

- **Nominal categorical variables** have no natural order.

  Examples: Eye colour, favourite brand of cereal, brand of car driven.

SUMMARY: TYPES OF VARIABLES

The different **types of variables** collected can be summarised as follows:



Warning!

It is not the variable name itself that determines whether the data is numerical or not. It is the way the data was recorded. Height, for example, could be recorded in metres and, hence, is numerical, BUT if height was recorded as 'tall', 'average' or 'short', it is ordinal categorical.

QUESTION 1
The level of water usage of 200 homes was rated in a survey as low, medium or high and the size of the houses as small, standard or large. The results of the survey are displayed in the table below.

Water Usage	House Size		
	Large	Standard	Small
Low	2	10	7
Medium	19	60	13
High	19	40	30

The variables, level of water usage and size of house, as recorded in this survey, are:

A Both nominal categorical variables.
B Both ordinal categorical variables.
C Nominal and ordinal categorical variables respectively.
D Continuous numerical and ordinal categorical variables respectively.
E Ordinal and nominal categorical variables respectively.

QUESTION 2
Which one of the following is an example of continuous numerical data?

A Number of runs made by a cricket player.
B Speed of a car captured by a speed camera.
C Your favourite secondary school year level.
D Shoe sizes.
E Labor / Liberal preference of 100 people surveyed.

QUESTION 3
30 shoppers using the express lane were surveyed to find the number of items being purchased. The results were as follows.

3 5 6 12 8 15 6 8 7 5 9 11 4 8 7 12 6 8 10 8 11 9 4 8 13 1 2 7 5 11 7

The type of data is:

A Discrete numerical.
B Continuous numerical.
C Ordinal categorical.
D Nominal categorical.
E Ordinal numerical.

VCAA EXAM QUESTION 1
(VCE EXAM 2 – 2022 – QUESTION 3A)

Table 1 displays data for seven weather-related variables for the first eight days of December 2021.

Table 1

Day number	*Minimum temperature* (°C)	*Maximum temperature* (°C)	*Rainfall* (mm)	*Maximum wind speed* (km/h)	*Direction of maximum wind speed*	*Temperature 9 am* (°C)
1	19.4	28.3	0	35	ENE	22.9
2	17.6	29.7	1.0	35	WSW	24.2
3	7.6	16.5	11.6	26	WSW	12.7
4	7.5	15.9	0	30	WSW	10.9
5	5.7	19.0	0.2	24	ESE	10.4
6	9.9	23.8	0	39	NE	17.8
7	11.0	11.9	0	22	SSW	11.7
8	6.5	14.2	0	28	ESE	9.5

Data: Commonwealth of Australia 2022, Bureau of Meteorology, <www.bom.gov.au>

Write down the number of numerical variables in Table 1. (1 mark)

UNIVARIATE ANALYSIS

The prime objectives of Univariate analysis are to determine:

- Type of distribution (i.e. shape and if any outliers)
- Central tendency
- Spread

ORGANISING DATA

Data needs to be organised so that further analysis can be completed promptly and accurately. The data is organised either as it is collected or afterwards.

The techniques used in General Maths are:

- Frequency Tables
- Dot plots
- Bar charts
- Histograms
- Stem and leaf plots
- Boxplots

In organising the data there will be the need to group the data.

- **Categorical data** is self–evident as each category would be labelled according to the category and is unique from the others.

- **Numerical continuous data** will often be grouped. No hard and fast rules apply, but generally the data should have between five and no more than fifteen groups. The size of the interval is usually in 1's, 2's, 5's or 10's with others as the need arise. To judge this correctly look at the highest and lowest value and divide by a suitable grouping interval to find the number of intervals so that there are no less than five groups or no more than 15 groups. Often the size of the interval is relevant in the "big picture".

- **Numerical discrete data** can be ungrouped if there are less than 15 different scores. If there are more, then usually it is grouped in multiples of 2, 5, 10 so there is a minimum of 5 groups and no more than 15 groups.

EXAMPLE 1
The following is a survey of 30 students recording the number of people in their family living at home. Summarise in a frequency table.

2 4 5 6 4 3 4 3 2 5 6 6 5 4 5 3 3 4 5 8 7 4 3 4 2 5 4 6 4 5

Solution

1. Identify the data as numerical discrete.

2. There are only 7 different scores so leave as ungrouped.

3. Use a frequency table of two (three) columns.

   The first column is for the possible scores (x) and can be labelled as Number of members in a family.

   The second column is for the frequency (f) and can be labelled Number of students or families. Optional 3rd column used as a tally for very large sets of data.

4. Add up the frequency (Σf) to confirm scores from 30 students has been recorded.

Number of People in the Family (x)	TALLY	Number of Families (f)
2		3
3		5
4		9
5		7
6		4
7		1
8		1
		$\Sigma f = 30$

VISUAL DISPLAYS

STEM PLOTS (STEM & LEAF PLOT)

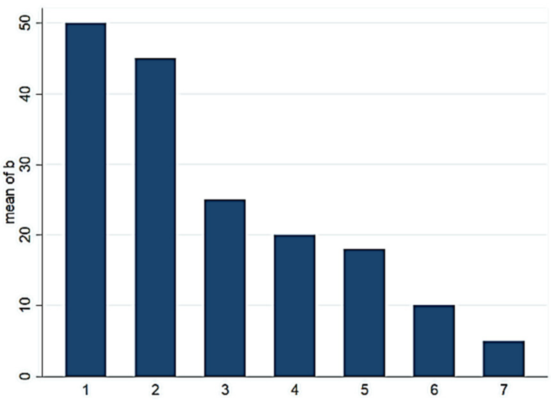
CD collection size of 20 students

Stem	Leaf
0	4
1	4 6
2	2 3 5 8
3	0 4 4 6 7 7 9
4	1 3 5 9
5	4 6

2 | 5 = 25 CDs

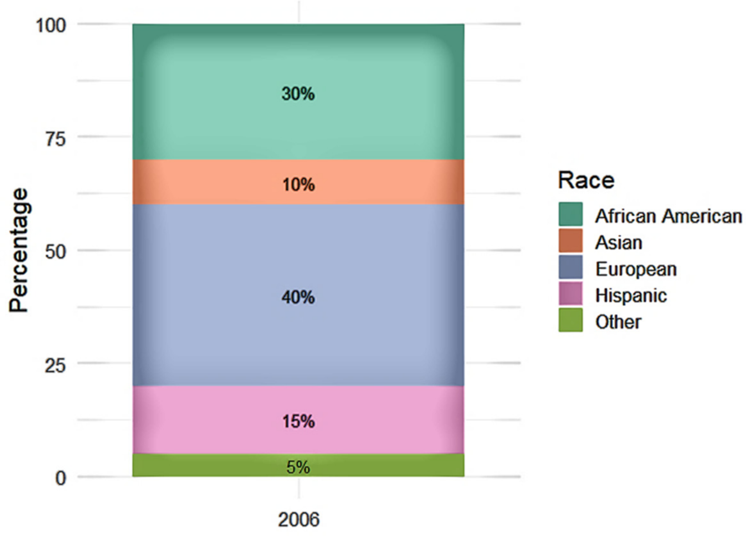
- Most useful technique for summarising small groups of numerical data.
- Data values summarised in neat columns with a legend/key
- From the stemplot shown the scores collected were 4, 14, 16, 22, 23, 25, 28, 30, 34, 34, 36, 37, 37, 39, 41, 43, 45, 49, 54, 56.
- All the original values are retained.
- Stemplots visually display the shape of the distribution.
- An ordered stem and leaf plot is more useful than an unordered one, as position of values in distributions can be easily found.

BAR GRAPHS



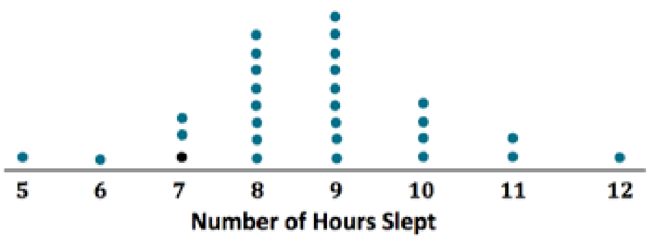
- For categorical data (or numerical discrete data).
- Length of each bar represents frequency or percentage frequency.
- Width of bars and spaces between bars must be kept uniform.
- Bars usually don't touch one another, although this is not a hard and fast rule.

SEGMENTED BAR CHARTS



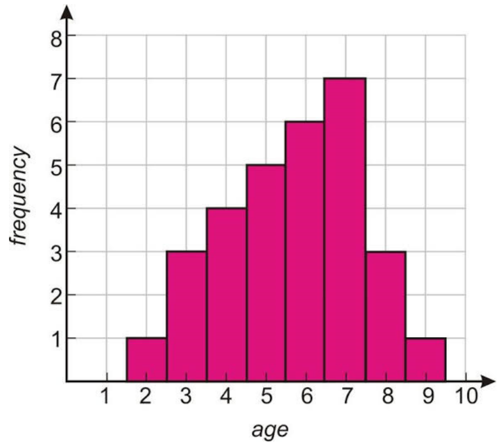
- For categorical data.
- A segmented bar chart is similar to a bar chart except the bars are stacked on one another to give a single bar.
- The lengths of the segments correspond to the data set frequency.
- Often percentage frequency is used. This is then called a percentaged segmented bar chart.
- The height of the bar gives the total frequency or 100% if percentage is used.

DOT PLOTS



- A dot plot is constructed by drawing a scale and then evenly placing dots above the appropriate values to represent the data points. There is no vertical axis
- Dot plots are graphical displays of discrete numerical or categorical data sets.
- A single dot represents each data point.
- The shape of the distribution is clear.
- They are often used for small data sets or to compare more than one data set.

HISTOGRAMS



- For numerical data (usually continuous grouped data) but used in some circumstances to display discrete data.
- The height of the bar gives the frequency (count or percentage).
- The end of one rectangle must be the beginning of the next rectangle i.e. no gaps.
- We can also plot percentage frequencies or relative frequencies on the vertical axis for both grouped and ungrouped data.
- If the data is individual discrete values, the bars are directly above the appropriate values.
- If the data is continuous class intervals the bars start at the bottom end of the class interval and extend to the upper end of the class interval.

QUESTION 4

The golf scores for 30 golfers were recorded as follows. Summarise as an ordered stem plot grouped in 5's. Comment on the shape of the distribution of scores.

63 72 74 71 70 75 66 73 69 75 79 68 85 78 76 72 73 71 70 81 73 74 82 87
78 79 68 75 76 72

Golf scores for 30 golf players:

Stem	Leaf
6	
6^5	
7	
7^5	
8	
8^5	

7 | 4 = 74 Strokes

QUESTION 5
For the stem and leaf plot shown:

Stem	Leaf
7	8
8	0 8 9
9	1 6 7 8
10	3 5 8
11	2

7 | 6 = 7.6

Which of the following data sets match the above stem & leaf plot?

A 78 80 88 89 91 96 97 98 103 105 108 112

B 8 0 8 9 1 6 7 8 3 5 8 2

C 7.8 8.0 8.8 8.9 9.1 9.6 9.7 9.8 1.03 1.05 1.08 1.12

D 7.8 8.0 8.8 8.9 9.1 9.6 9.7 9.8 10.3 10.5 10.8 11.2

E 8.8 8.9 9.1 9.6 9.7 9.8 10.3 10.5 10.8

QUESTION 6
Display the following data set as a

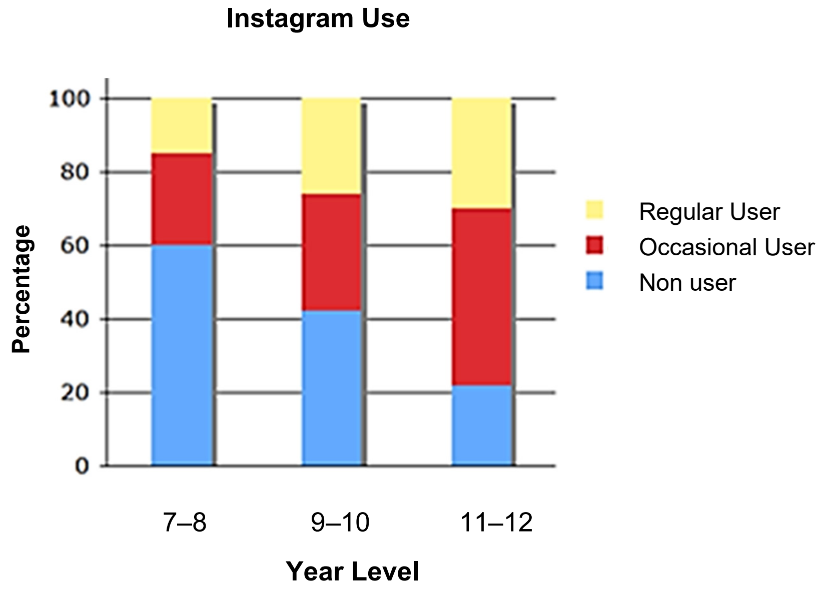
(i) bar chart
(ii) dot plot

Type of sport played by 32 students on the weekend:

basketball football basketball soccer tennis netball basketball football football tennis netball netball football soccer netball karate football football athletics basketball netball tennis football soccer football soccer basketball athletics soccer tennis soccer hockey

For Barcharts: 1. Identify all the different categories. 2. Give the barchart a suitable title. 3. Label the horizontal axis with the names of the categories of sport. Leave a space between each category. 4. Label the vertical axis as frequency or number of students. 5. Count the number of each sport and make the bar the height of that frequency.	**Follow the instructions provided.**
For Dot Plots: 1. Identify all the different categories. 2. Give the dot plot a suitable title. 3. Label the horizontal axis with the names of the categories of sport. Leave a space between each category. 4. Use a single dot to represent each data point.	

QUESTION 7

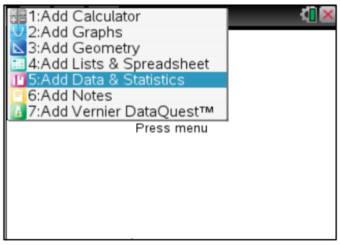
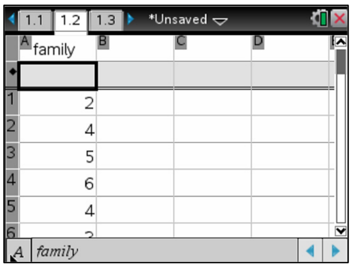
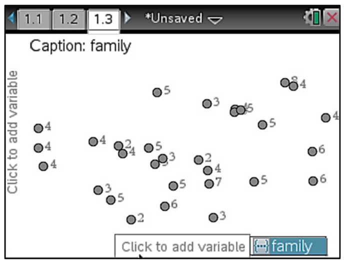
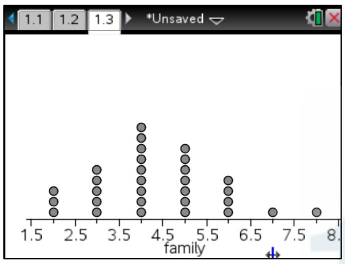
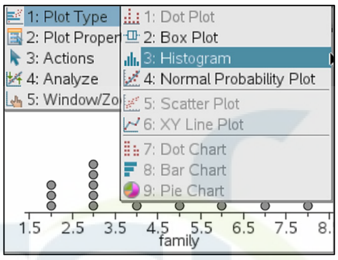
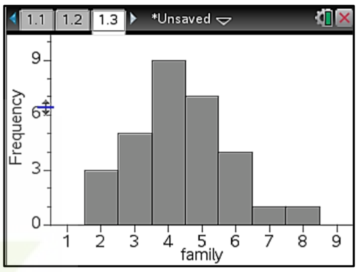


The segmented bar chart shows the distribution of Instagram use by a random sample of school students. For these students, the percentage in year levels 9 – 10 who are 'occasional Instagram users' is closest to:

A 25%
B 32%
C 42%
D 50%
E 75%

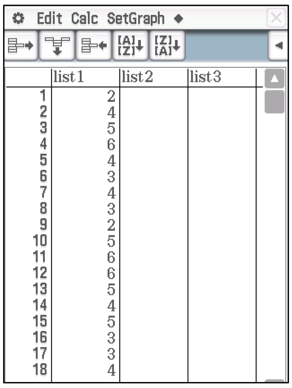
USING THE TI–NSPIRE CALCULATOR

<table>
<tr>
<td>Control → I → Menu → 4. Add Lists & Spreadsheet

Control → I → Menu → 5. Add Data & Statistics

</td>
<td>Enter data into Lists and spreadsheet, give column a name here “family”

</td>
<td>Go to Data & Statistics, click to add variable “family”

</td>
</tr>
<tr>
<td>The display will automatically be a dot plot

</td>
<td>Menu → 1. Plot type → 3. Histogram

</td>
<td>Histogram is displayed

</td>
</tr>
</table>

USING THE CASIO CLASSPAD CALCULATOR

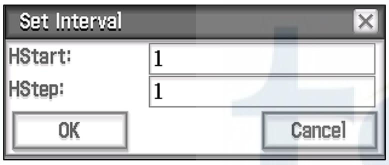
In Statistics menu, enter data into List 1



SetGraph → Setting → set screen as shown → Set → 

Set the interval, here we want to start at 1 and go up in class intervals of 1 → OK

The graph will appear as shown in the bottom half of the screen.



Resize screen if necessary, Analysis → trace will allow you to find heights of peaks using the 4 way key to move across.