



HOW WELL DO YOU KNOW YOUR COURSE MATERIALS?

These questions (and many others) will be addressed in detail in the TSFX “Unit 3 Exam Revision Lectures” in the Term 3 school holidays.

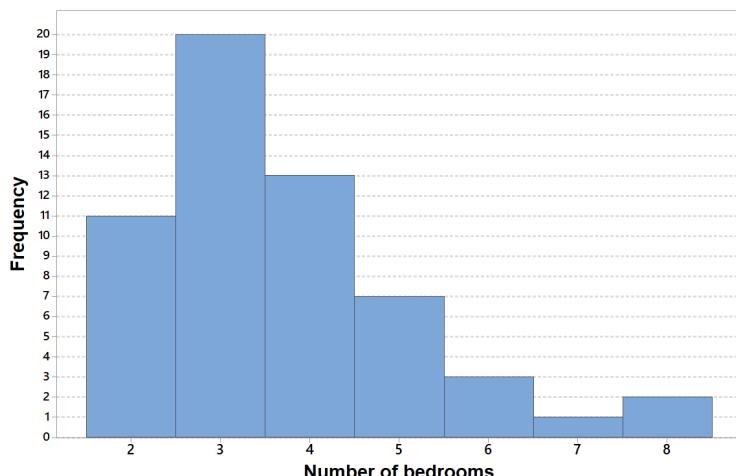
UNIT 3 FURTHER MATHEMATICS CORE – DATA ANALYSIS

Question 1

The mean score for a test for 9 students in a Further Maths class is 65%. Another student does the test and the mean score increases to 68%. What score does the last student get in the test?

Question 2

The display below shows the number of bedrooms in 57 houses advertised for sale during March 2018.



The median number of bedrooms for these houses is:

- A. 3
- B. 4
- C. 5
- D. 6
- E. 7

Question 3

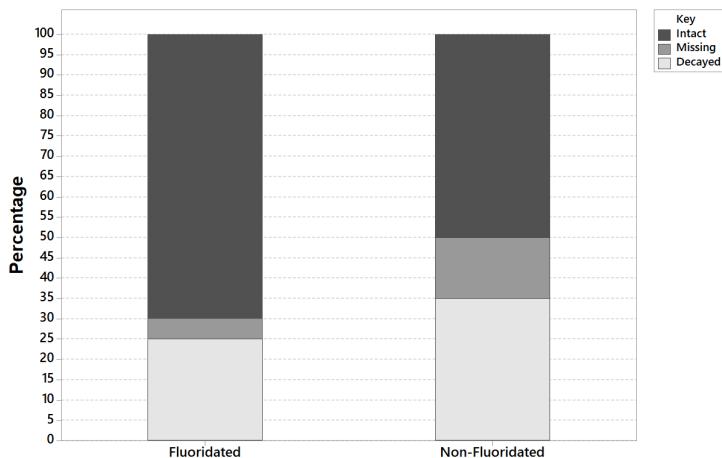
In a particular data set the maximum value 86 is an outlier, but the value 84 is not. The minimum value in the data set is 58 and this value is not an outlier.

Which of the following statements is not true about the data set?

- A. The data distribution must be positively skewed.
- B. The IQR could be 11, but could not be 12.
- C. The minimum could be the same as Q_1 .
- D. The median could be the same as Q_3 .
- E. The IQR could not be 6 or less.

Question 4

The segmented bar charts display the average percentages of decayed, missing and intact teeth of people living in regions with fluoridated and non-fluoridated water.



Do the percentage segmented bar charts support the contention that fluoridation of water is associated with dental health? Explain your answer with reference to appropriate percentages.

Question 5

The relationship between a student's score in their English exam and their score in a Further Maths exam is given by the least squares regression equation

$$\text{Further Maths score} = 3.75 + b \times \text{English score}.$$

If the average score for English is 65% and the Further Maths score is 72% and the standard deviation for Further Maths is 50% greater than the standard deviation for English, what percentage of the variation in the Further Maths score can be explained by the variation in the English score?

Question 6

The ATP rankings and earnings of the top 13 male tennis players is given in the table below:

Ranking	1	2	3	4	5	6	7	8	9	10	11	12	13
Earnings (\$millions)	4.3	2.8	2.3	1.9	1.9	1.9	1.4	1.2	1.1	1.1	1.1	1	0.9

A reciprocal transformation of the ranking axis is performed. Write the equation of the least squares regression line for the transformed data with the slope and intercept written correct to three significant figures.

Question 7

The seasonal indices for sales of sunglasses at a particular shop are shown below:

Summer	Autumn	Winter	Spring
1.575	0.750	0.625	1.05

To correct for seasonality for sales during Winter, the sales would be

- A. Increased by 37.5%
- B. Increased by 62.5%
- C. Increased by 60%
- D. Decreased by 37.5%
- E. Decreased by 62.5%

CORE – RECURSION AND FINANCIAL MODELLING

Question 8

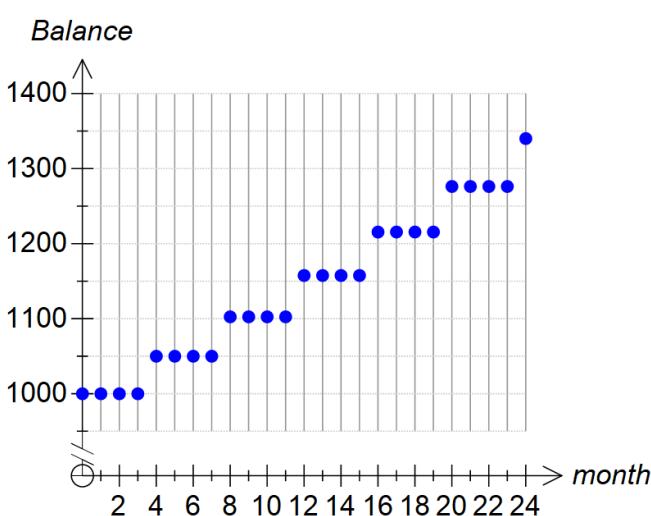
The recurrence relation $A_0 = 15000$ $A_{n+1} = 1.0075 A_n - 690$ models the balance of a reducing balance loan after n months. What is the total interest that would be paid on this loan, if the loan is reduced to zero with a final adjusted payment?

Question 9

Olivia takes out a loan of \$300 000 to buy a house. She will pay off the loan with equal monthly payments over a 20 year period at 4.1% per annum compounding monthly. After 10 years, what is the percentage of the principal that she has paid off her loan correct to the one decimal place?

Question 10

The graph below shows the balance of an investment over a 24 month period:



The graph represents an

- A. Investment which attracts interest compounding monthly
- B. Investment which attracts interest compounding quarterly
- C. Investment which attracts simple interest added monthly
- D. Investment which attracts simple interest added quarterly
- E. Investment which attracts interest compounding annually

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ANSWERS

Question 1

Answer: 95%

$$\frac{65 \times 9 + x}{10} = 68, x = 95$$

Question 2

Answer is A.

There are 57 houses in the survey, so the median value is the 29th value. The 12th to 31st values are 3, so 3 is the median.

Question 3

Answer is E.

Answer A is incorrect because skewing cannot be determined from the information given. If the IQR was small the data set could have a negative skew, if the IQR was larger the data could be positively skewed, but in each case it would be determined by the actual position of the median in relation to each minimum and maximum.

Answer B is correct because if $IQR = 11$, $1.5 \times IQR = 16.5$. Because 86 is an outlier it is more than $Q_3 + 16.5$ so that means that $Q_3 < 86 - 16.5 < 69.5$ and because the $IQR = 11$, $Q_1 < 69.5 - 11 < 58.5$. As $\text{Min} \leq Q_1$ and $58 \leq 58.5$, it is possible that $IQR = 11$.

On the other hand, if $IQR = 12$, $1.5 \times IQR = 18$, that means that $Q_3 < 86 - 18 < 68$ and because the $IQR = 12$, $Q_1 < 68 - 12 < 56$. As $\text{Min} \leq Q_1$ and $58 > 56$, it is not possible that $IQR = 12$.

Answers C and D are both correct because the five figure summary values are all positional, so if there was a large number of values that were the same, either of these could be true.

Answer E is correct because if $IQR = 6$, then $Q_3 + 1.5 \times 6 < 86$ so $Q_3 < 77$ and $Q_1 = Q_3 - 6 < 71$ and any value more than $1.5 \times IQR$ below Q_1 would be an outlier. $58 < 71 - 1.5 \times 6 < 62$ so it would be an outlier. Even if 84 was on the upper fence, the lower fence would be $2 \times 9 + 6$ less than 84, so it would be 60 and 58 would still be an outlier.

Question 4

Yes because the percentage of decayed teeth is less in fluoridated water regions at 25% than in non-fluoridated water regions at 35%.

OR

Yes because the percentage of missing teeth is less in fluoridated water regions at 5% than in non-fluoridated water regions at 15%.

OR

Yes because the percentage of intact teeth is more in fluoridated water regions at 70% than in non-fluoridated water regions at 50%.

THE SCHOOL FOR EXCELLENCE

Question 5

Answer: 49%

Using $y = a + bx$, $b = \frac{r \times S_y}{S_x}$ and $a = \bar{y} - b \times \bar{x}$:

$$a = 3.75 = 72 - b \times 65 \quad b = 1.05$$

$$1.05 = \frac{r \times S_y}{S_x} = 1.5 r \quad (\text{you can use } 1.5 \text{ for } \frac{S_y}{S_x} \text{ as } S_{FM} \text{ is } 50\% \text{ greater than } S_E)$$

Using your solver $r = 0.7$. The coefficient of determination is $r^2 = 0.7^2 = 0.49$, so 49% of the variation in the Further Maths score can be explained by the variation in the English score

Question 6

Answer: Earnings in millions = $0.876 + \frac{3.62}{\text{ranking}}$

Linear Reg

$y=a+b \cdot x$

a	=0.8757752
b	=3.6208923
r^2	=0.9743536
MSe	=0.9493649
	=0.0504085

OK

Question 7 Answer is C.

Correcting for seasonality means dividing by the seasonal index of 0.625. Dividing by 0.625 is the same as multiplying by the reciprocal of 0.625 = $\frac{1}{0.625} = 1.6$. This is equivalent to an increase of 60%.

Question 8 Answer: \$1436.16

Compound Interest	Compound Interest	Compound Interest																																										
<table border="1"><tr><td>N</td><td>23</td></tr><tr><td>I%</td><td>9</td></tr><tr><td>PV</td><td>15000</td></tr><tr><td>PMT</td><td>-690</td></tr><tr><td>FV</td><td>0</td></tr><tr><td>P/Y</td><td>12</td></tr><tr><td>C/Y</td><td>12</td></tr></table>	N	23	I%	9	PV	15000	PMT	-690	FV	0	P/Y	12	C/Y	12	<table border="1"><tr><td>N</td><td>23</td></tr><tr><td>I%</td><td>9</td></tr><tr><td>PV</td><td>15000</td></tr><tr><td>PMT</td><td>-690</td></tr><tr><td>FV</td><td>-561.9436593</td></tr><tr><td>P/Y</td><td>12</td></tr><tr><td>C/Y</td><td>12</td></tr></table>	N	23	I%	9	PV	15000	PMT	-690	FV	-561.9436593	P/Y	12	C/Y	12	<table border="1"><tr><td>N</td><td>1</td></tr><tr><td>I%</td><td>9</td></tr><tr><td>PV</td><td>-561.9436593</td></tr><tr><td>PMT</td><td>566.1582367</td></tr><tr><td>FV</td><td>0</td></tr><tr><td>P/Y</td><td>12</td></tr><tr><td>C/Y</td><td>12</td></tr></table>	N	1	I%	9	PV	-561.9436593	PMT	566.1582367	FV	0	P/Y	12	C/Y	12
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There will be 23 equal payments of \$690.	After 23 payments \$561.94 is still owed.	The final payment is \$566.16. The total paid is $690 \times 23 + 566.16 = \$16\,436.16$. The total interest is $16436.16 - 15000 = \$1436.16$																																										

Question 9 Answer: 39.9%

Compound Interest		Compound Interest		
N	240	N	120	She has paid off 300000 –
I%	4.1	I%	4.1	180275.77 = \$119 724.23
PV	300000	PV	300000	
PMT	-1833.787814	PMT	-1833.79	
FV	0	FV	-180275.767	$\frac{119724.23}{300000} \times 100 = 39.9080\ldots\%$
P/Y	12	P/Y	12	
C/Y	12	C/Y	12	

She needs to pay \$1833.79 per month.

After 10 years she still owes \$180 275.77

Answer is 39.9%

Question 10 Answer is B.

The balance is recorded monthly but increases every quarter due to interest. The amount added each quarter is gradually increasing and so this is compound interest.