

# UNIT 3 SPECIALIST MATHEMATICS

## BOOK 1

### Complex Numbers

The Imaginary Number  
Operations Involving Imaginary Numbers

### Properties of Complex Numbers

Equality  
Addition and Subtraction  
Multiplication  
Complex Conjugates  
Magnitude  
The Multiplicative Inverse  
Division of Complex Numbers

### The Complex Number Plane

Geometrical Interpretation of Subtraction  
Polar Form  
Converting Cartesian Forms into Polar Form  
Converting the Polar Form into Cartesian Form  
Multiplication and Division in Polar Form  
Geometrical Interpretation of Multiplication and Division

### De Moivre's Theorem

Solving Equations in the Form  $z^n = a$  Using De Moivre's Theorem  
Finding  $n$ th Roots of a Complex Number  
The  $n$ th Roots of Unity  
Finding Square Roots in Exact Cartesian Form

### Polynomials Over $\mathbb{C}$

The Fundamental Theorem of Algebra  
The Factor Theorem  
The Conjugate Root Theorem

### Factors Over $\mathbb{C}$ of Polynomials

Factorising Quadratics Over  $\mathbb{C}$   
Factorising Cubics Over  $\mathbb{C}$   
Factorising Polynomials of Degree Greater Than 3 Over  $\mathbb{C}$

### Solution Over $\mathbb{C}$ of Polynomial Equations

### Relations and Regions in the Complex Plane

#### Relations in the Complex Plane

Restrictions on Magnitude  
Solving Questions Involving Restrictions on Magnitude  
Rays and Lines  
Common Types of Relations – Summary

#### Regions in the Complex Plane

Common Types of Regions – Summary

## **BOOK 2 – VECTORS**

### **Vectors**

Vector Notation  
The Negative Vector  
Position Vectors  
Equality of Vectors  
Addition of Vectors  
The Identity Vector  
Subtraction of Vectors  
Multiplication of a Vector by a Scalar  
Unit Vectors (Definition)  
Vectors in 3-Dimensional Space  
The Magnitude (Size) of a Vector  
The Magnitude (Size) of Vectors Multiplied by Scalars

### **Vector Algebra in Component Form**

Vector Equivalence

### **The Distance Between Two Vectors**

#### **Unit Vectors**

Creating Unit Vectors

#### **Direction Cosines**

#### **The Scalar Product of Two Vectors**

Properties of the Scalar Product  
The Angle Between Vectors vs The Dot Product  
The Scalar Product of Vectors in Component Form  
Angles Between Vectors

#### **Resolving Vectors**

Linear Projections  
Identifying Projections  
The Scalar Resolute/Projection  
Perpendicular Scalar Resolutes  
Vector Resolutes  
Perpendicular Vector Resolutes

#### **Linear Dependence and Independence**

#### **Vector Proofs**

A General Approach to Vector Proofs  
Proofs Involving Circles  
Proofs Involving Lines  
Proofs Involving Pyramids  
Proofs Involving Quadrilaterals  
Proofs Involving Parallelograms  
Proofs Involving Trapezia  
Proofs Involving Triangles

## BOOK 3 – CALCULUS I

### Differential Calculus

Formula List

The Derivative of  $\tan(kx)$  and  $\cot(kx)$

The Second Derivative

Applications of the Second Derivative

Implicit Differentiation

Derivatives of Inverse Circular Functions

### Integral Calculus – Techniques in Anti-Differentiation

Definition

Basic Properties

Standard Anti-Derivatives

Linear Substitution

The 'Reverse Chain Rule'

Anti-Derivatives of  $\frac{1}{\sqrt{a^2 - x^2}}$  and  $\frac{1}{a^2 + x^2}$

Anti-Derivatives of  $\sin^2(kx)$  and  $\cos^2(kx)$

Anti-Derivatives of odd powers of  $\sin(kx)$  and  $\cos(kx)$

Anti-Derivatives of  $\tan^n(kx)$  and  $\cot^m(kx)$

Anti-Derivatives of Expressions of the Form  $\sin^m(kx)\cos^n(kx)$

Partial Fraction Decomposition

Rational Functions

$N(x)$  = Polynomial of Degree 2 or Higher

Anti-Derivatives of Rational Functions with Quadratic Denominators

Anti-Differentiation by Recognition

The Relationship between the Graph of a Function and the Graph of its Anti-Derivative

### Integral Calculus – Applications in Integration

Definite Integrals

Basic Properties

The Area Under a Curve

The Area Between a Curve and the Y Axis

The Area Between Two Curves

Volumes of Solids of Revolution

The Volume Between Two Curves

### Lengths of Curves in the Plane

The Length of a Parametric Curve

## BOOK 4

### Area of Study 1: Functions and Graphs

#### Section 1: Rational Power Functions of Low Degree

Asymptotes

Sketching Rational Functions

Graphs of Power Functions

Graphing Rational Functions Using the Addition of Ordinates

Graphing Rational Functions Using Multiplication of Ordinates

Graphing Rational Functions Using the Key Features on a Graph

Graphing Rational Functions Using the Reciprocal Theory

Graphing Rational Functions Using Partial Fractions  
Sketching Rational Functions – Executive Summary

## Section 2: Conics

Equations Describing Conics

The Circle

The Ellipse

Standard Form of an Ellipse

Sketching Graphs of Ellipses

Expanded Form of an Ellipse

Writing Equations of Ellipses

The Non-Rectangular Hyperbola

Standard Form of the Non-Rectangular Hyperbola (Centre (0, 0))

Sketching the Hyperbola

Standard Form of the Non-Rectangular Hyperbola (Centre (h, k))

The Conjugate Hyperbola

Important Features of Hyperbolae

Expanded Form of a Hyperbola

Writing Equations of Hyperbolae

## Section 3: The Absolute Value (Modulus) Function

Properties of the Modulus Function

Converting a Modulus Inequation to an Interval and Visa Versa

Graphing Modulus Functions

The Graph of  $y = |x|$

The Graph of  $y = |f(x)|$

Sketching Modulus Functions in the Form  $y = |f(x)|$

Sketching Modulus Functions in the Form  $y = a|f(x-h)| + k$

Modulus Functions in the Form  $y = f(|x|)$

Sketching Modulus Functions in the Form  $y = f(|x|)$

Writing Modulus Functions as Hybrid Functions

Sketching Modulus Functions Expressed as Hybrid Functions

Solving Modulus Equations and Inequations

The Algebraic Approach

The Graphical & Algebraic Approach

Solving Equations in the Form  $|x| = |b|$

## Section 4: Circular (Trigonometric) Functions

Reciprocal Trigonometric Functions

Graphs of Reciprocal Trigonometric Functions

The Fundamental Identities

The Addition Theorems

The Double Angle Formulae

Graphs of Inverse Trigonometric Functions

Maximal Domains and Ranges

Transformations of Functions – Summary