UNIT 3 MATHEMATICAL METHODS

BOOK 1 – ALGEBRA & MATRICES

Section 1: Algebra of Polynomial and Algebraic Functions

Substitution of Values
Rearrangement and Substitution

Polynomial Expressions
Expanding Expressions
Expanding Expressions by Rule
- Perfect Squares
- The Difference of Two Squares
- Expanding Perfect Cubes by Rule

Techniques in Factorisation
Number Systems
Highest Common Factors
The Difference of Two Squares
Quadratic Trinomials
- Factorising Quadratic Trinomials by Rule
- Factorising Quadratic Trinomials (FOIL)

Equations That Can Be Reduced to Quadratic Expressions
Completing the Square
Transposition Using Completing the Square
The Sum or Difference of Two Cubes
Grouping Terms
Long Division of Polynomial Expressions
Division Done Easy
Synthetic Division
The Remainder Theorem
The Factor Theorem
Finding Factors
Factorising Higher Order Polynomial Expressions

Solving Algebraic Equations
Verifying Solutions
The Quadratic Formula
The Discriminant
Solving Equations by Equating Coefficients
Advanced Algebraic Applications
Numerical Estimation of Roots/Solutions
Literal Equations

Simultaneous Equations
Solving Linear Equations (Two Variables)
Predicting Solutions of Linear Equations by Inspection
Solving Literal Linear Simultaneous Equations
Solutions of Literal Linear Simultaneous Equations
Solving One Linear and One Second Degree Equation

Section 2: Matrices

Matrices
Dimension/Order/Size of Matrices
Representing Matrices and Elements
Special Types of Matrices

Equality of Matrices

Algebra of Matrices
Addition and Subtraction of Matrices
Scalar Multiplication
Matrix Multiplication
The Identity Matrix
The Determinant & Multiplicative Inverse (Inverse Matrix)
Finding Inverse Matrices

Solving Matrix Equations
Solving Linear Simultaneous Equations in Terms of Two Variables
Solutions of Simultaneous Linear Equations
Simultaneous Equations in Terms of Three Unknowns
Geometric Representation of Solutions to Equations With 3 Unknowns
Simultaneous Equations with More than 3 Unknowns

Section 3: QUESTIONS FROM PAST VCE EXAMINATION PAPERS
Past VCE Exam Questions

BOOK 2 – RELATIONS, FUNCTIONS & THEIR GRAPHS

Relations, Functions and Their Graphs
Relations
Types of Relations
Functions and Inverses

Function Notation
Number Systems
Notations Used to Describe Domains and Ranges
The Domain
The Range
The Largest Possible Domain
Continuity and Discontinuity

Curve Sketching
Common Curve Shapes
Determining the Sign of a Relation
Sketching Curves by Considering the Essential Features of a Graph
- Y Intercepts
- X Intercepts
- Stationary Points
- Asymptotes
- Asymptotic Behaviour

Sketching and Solving Inequations

Relations and Transformations
Dilations
Reflections
Translations
Summary of Transformation Notations
The Order of Transformations
Identifying Transformations
Transformation Mapping Notation
Matrix Representation of Transformations
- Finding the Image of a Point
- Finding the Image of a Relation of Function

Polynomial Functions
A. Linear Functions and Their Graphs
B. Quadratic Functions and Their Graphs
C. Cubic Functions and Their Graphs
D. Quartic Functions and Their Graphs
E. Higher Order Polynomial Functions

Negative Power Functions
A. The Rectangular Hyperbola
Higher Order Negative Odd Power Functions
Transformations Involving Negative Odd Power Functions
B. The Truncus
Higher Order Negative Even Power Functions
Transformations Involving Negative Even Power Functions

Fractional Power Functions
Square Root Functions and Their Graphs
Other Fractional Power Functions
Transformations Involving Square Root and Related Functions

The Discriminant Function
Calculating the Discriminant
Using the Discriminant in Proofs Involving X Intercepts or Roots
Other Applications Involving the Discriminant

BOOK 3

Section 1: Exponential & Logarithmic Functions

Exponential and Logarithmic Functions
Indicial Expressions and Equations
Rational Exponents

Index Laws
Simplifying Indicial Expressions

Solving Indicial Equations

Logarithms
Simplifying Logarithmic Expressions
Logarithmic Laws
Solving Logarithmic Equations
Change of Base Rule

Review of Transformations
Transformation Notations
Identifying Transformations
Matrix Representation of Transformations
Finding the Image of a Function or Relation

Graphing Exponential and Logarithmic Functions
The Exponential Graph
Transformations Involving Exponential Functions
Solving Exponential Inequations
Finding Equations Describing Exponential Functions
The Logarithmic Graph
Transformations Involving Logarithmic Functions
Finding Equations Describing Logarithmic Functions
Solving Logarithmic and Exponential Inequations

Modelling with Exponential Functions

Modelling with Logarithmic Functions

Section 2: Circular Functions (Trigonometry)

Circular Functions/Trigonometry
Circles and Trigonometric Ratios
The Unit Circle
Angle Measures
Angle Conversions
Angle Directions
Quadrants and Angles
Summary of Signs (CAST)
Angles on the Axes
Exact Values Based on the Axes
Inverse Operations
Reciprocal Functions
Exact Values in the First Quadrant
Symmetry Properties of the Unit Circle
- Supplementary Angles
- Negative Angles
- Complementary Angles

Exact Values in Other Quadrants
Summary – Exact Values to Commit to Memory
Summary – Rules to Commit to Memory
Trigonometric Identities

Graphs of Trigonometric Functions
The Sine Graph
The Cosine Graph
The Tangent Graph
The Relationship Between Sine and Cosine
The Amplitude
The Period
Horizontal Translations
Vertical Translations
The Range (Maximum and Minimum Values)
Asymptotes
Transformations Involving Trigonometric Functions
Sketching Trigonometric Functions by Considering Transformations
Determining Rules for Sine and Cosine Functions
Determining Rules for Tangent Functions

Solving Trigonometric Equations
The Significance of Solving Trigonometric Equations
Solving Trigonometric Inequations
Solving Complex Trigonometric Equations
General Solutions for Trigonometric Equations

The Period of Combined Functions

Extended Response Style Questions

BOOK 4

Section 1: Techniques in Differentiation

Limits and Derivatives
Conditions For the Existence of a Limit
Evaluating Limits Graphically
Evaluating Limits Algebraically
Limit Theorems
Differentiation
Derivatives From First Principles

Differentiation by Rule
Derivatives of Polynomial and Rational Functions
Finding Derivatives – Method
Derivatives of Trigonometric Functions
Derivatives of Logarithmic Functions
Derivatives of Exponential Functions

The Chain Rule
Differentiating Functions Involving Brackets and Powers
Differentiating Complex Trigonometric Functions
Differentiating Complex Exponential Functions
Differentiating Complex Logarithmic Functions

The Product Rule

The Quotient Rule

Summary of Differentiation Techniques

Conditions For Differentiability

Gradients at Specific Points

Derivatives of Hybrid Functions

Derivatives of Composite Functions

Section 2: Applications in Differentiation

Graphs of the Derivative Function
Sketching the Derivative Function
Sketching $f(x)$ from the Derivative Function

The Newton-Raphson Method

Joining Functions Smoothly

Stationary Points
Locating Stationary Points
The First Derivative Test
The Second Derivative Test
False Stationary Points
Maxima and Minima
Maximum and Minimum Values
Solving Worded Optimisation Questions

Increasing and Decreasing Functions

Strictly Increasing and Strictly Decreasing Functions

Tangents and Normals

Rates of Change
Rates and Graphs
Vessels and Rates of Change
Average and Instantaneous Rates of Change
Solving Worded Problems Involving Rates of Change
Motion in a Straight Line
Relationships Between Displacement, Velocity and Acceleration

Analysis Tasks