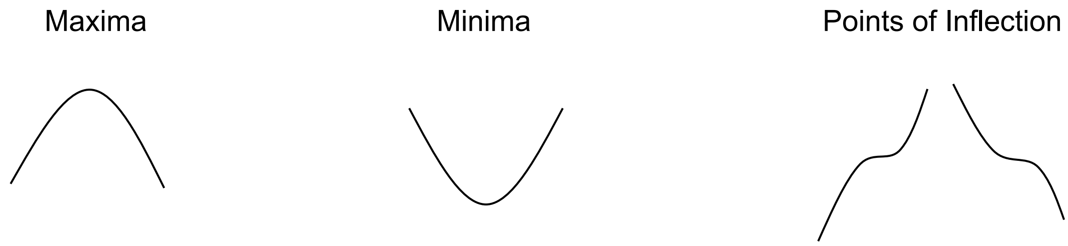
CONTENTS

STATIONARY (CRITICAL) POINTS

Stationary points occur at regions where the gradient of the tangent to the curve is equal to zero. i.e. $\frac{dy}{dx} = 0$.

Examples of stationary points include:



Maximum stationary points are referred to as local maxima as these points are positioned higher or above the points on either side. Notice that the gradient on the immediate left of this point is positive and on the immediate right, it is negative. This pattern in gradients is a characteristic feature of maximum stationary points.

Minimum stationary points are referred to as local minima as these points are lower than the other points within the local or immediate area. Notice that the gradient on the immediate left of the point is negative, and immediately to the right, the gradient is positive. This pattern in gradients is a characteristic feature of minimum stationary points.

Stationary points of inflection occur at regions where the sign of the gradient does not change on either side of the stationary point.

LOCATING STATIONARY POINTS

Step 1: Find the derivative.

Step 2: Let the derivative equal zero, and solve for x. These values of x represent the x coordinates of the stationary points.

Step 3: Find the corresponding value(s) of y by substituting the values of x into the equation describing the curve.

Step 4: Determine the nature of the stationary point(s) by observing the changes in the sign of the gradient (the First Derivative Test).

THE FIRST DERIVATIVE TEST

The First Derivative Test is used to determine the nature of a stationary point. This test requires that we observe the changes in the sign of the gradient on either side of each stationary point.

Step 1: Determine the x coordinate(s) of the stationary points.

Step 2: Select a value of x on either side of each stationary point, making sure that these values of x do not cross over into the other stationary point's domain.

Step 3: Substitute the value(s) of x into the derivative equation **and state** the numeric answer(s) obtained.

Step 4: Use the changes in the signs of the derivatives to determine the nature of each stationary point.

If the derivative changes from a positive value (on the left hand side of a stationary point) to a negative value (on the right hand side of the stationary point), then the point is a **maximum turning point.**

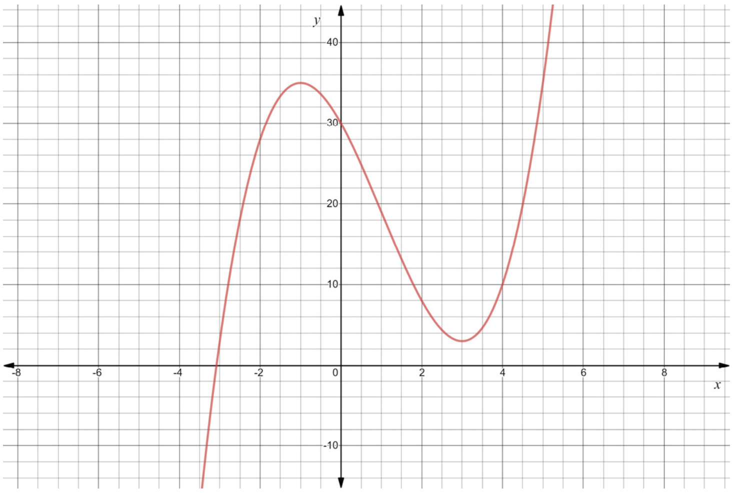
If the derivative changes from a negative value (on the left hand side of a stationary point) to a positive value (on the right hand side of the stationary point), then the point is a **minimum turning point.**

If the sign of the derivative does not change on either side of the stationary point, then that point is referred to as a **stationary point of inflection**.

QUESTION 49

Consider the curve whose equation is $f(x) = x^3 - 3x^2 - 9x + 30$. Find the coordinates of the stationary point(s) and determine their nature using an appropriate calculus technique.

Solution



$y = x^3 - 3x^2 - 9x + 30$

Find the derivative:

$$f(x) = x^3 - 3x^2 - 9x + 30$$
$$\therefore f'(x) = 3x^2 - 6x - 9$$

Let the derivative equal zero, and solve for x:

$$3x^2 - 6x - 9 = 0$$
$$x^2 - 2x - 3 = 0$$
$$(x-3)(x+1) = 0$$

$$\therefore x = -1, 3$$

Find the corresponding values of y:

Substitute $x = -1$ and $x = 3$ into $f(x)$.

$$f(-1) = (-1)^3 - 3(-1)^2 - 9(-1) + 30 = 35 \qquad \therefore (-1, 35)$$
$$f(3) = (3)^3 - 3(3)^2 - 9(3) + 30 = 3 \qquad \therefore (3, 3)$$

Determine the nature of the stationary point(s):

Using the First Derivative Test (FDT):

x	-2	-1	0	3	4
$f'(x)$	15	0	-9	0	15
Shape	/	—	\	—	/

As the gradient changes from positive to negative, $x = -1$ represents a maximum stationary point.

As the gradient changes from negative to positive, $x = 3$ represents a minimum stationary point.

QUESTION 50

Sketch the graph of the curve $y = x^4 - 2x^2 + 1$ for $-2 \le x \le 3$, labelling the coordinates of the axial intercepts and stationary points.

Solution

QUESTION 51

(a) Show that the stationary points on the curve $y = \frac{(1-x)^3}{(1-2x)}$ lie at $x = 1$ and $x = \frac{1}{4}$.

(b) Prove the nature of these points using an appropriate calculus technique.

Solution