MIXED TECHNIQUES IN DIFFERENTIATION
WORKSHEET 2

Differentiate the following equations with respect to x.

(a) $f(x) = 2\sin(3x)\cos(4x)$

(b) $f(x) = \dfrac{1+\ln x}{2+3e^{x}}$

(c) $f(x) = x\cos x$

(d) $f(x) = \left(xe^{x}\right)^{\pi}$

(e) $f(x) = \dfrac{1}{x^{0.4}}$

(f) $f(x)=\tan\left(2\pi x+\frac{\pi}{2}\right)$

(g) $f(x)=\frac{1}{\left(3x^4+x-8\right)^9}$

(h) $f(x)=e^x\left(\log_e x\right)^2$

(i) $f(x)=\left(x^4+x\right)^{(2x-1)}$

(j) $f(x)=3e^x-7\log_e x$

(k) $f(x)=\left(2x-1\right)^3\left(x^2-3\right)^2$

(l) $f(x)=\sqrt{x}\sin x+\cos x$

(m) $f(x)=\dfrac{2x-\sqrt{x}}{3-x}$

SOLUTIONS

(a) $f(x) = 2\sin(3x)\cos(4x)$

$$f(x) = 2\sin(3x)\cos(4x)$$

$$f'(x) = 2\sin 3x \times \frac{d}{dx}(\cos 4x) + \cos 4x \times \frac{d}{dx}(2\sin 3x)$$

$$= (2\sin 3x \times -4\sin 4x) + (\cos 4x \times 6\cos 3x)$$

$$= -8\sin(3x)\sin(4x) + 6\cos(3x)\cos(4x)$$

(b) $f(x) = \dfrac{1+\ln x}{2+3e^x}$

$$f(x) = \frac{1+\log_e x}{2+3e^x}$$

$$f'(x) = \frac{(2+3e^x)\frac{d}{dx}(1+\log_e x) - (1+\log_e x)\frac{d}{dx}(2+3e^x)}{(2+3e^x)^2}$$

$$= \frac{(2+3e^x)\left(1+\frac{1}{x}\right) - (1+\log_e x)(3e^x)}{(2+3e^x)^2}$$

(c) $f(x) = x\cos x$

$$f(x) = x\cos x$$

$$f'(x) = x \times -\sin x + \cos x$$

$$= -x\sin x + \cos x$$

(d) $f(x)=\left(xe^{x}\right)^{\pi}$

$$f(x)=\left(x e^{x}\right)^{\pi}=\left(x^{\pi}\right)\left(e^{\pi x}\right)$$

$$f'(x)=\left(x^{\pi}\times \pi e^{\pi x}\right)+\left(e^{\pi x}\times \pi x^{\pi-1}\right)$$

$$=\pi x^{\pi} e^{\pi x}+\pi x^{\pi-1} e^{\pi x}$$

$$=e^{\pi x}\left(\pi x^{\pi}+\pi x^{\pi-1}\right)$$

(e) $f(x)=\dfrac{1}{x^{0.4}}$

$$f(x)=\frac{1}{x^{0.4}}=x^{-0.4}$$

$$f'(x)=-0.4x^{-1.4}$$

$$=-\frac{0.4}{x^{1.4}}$$

(f) $f(x)=\tan\left(2\pi x+\dfrac{\pi}{2}\right)$

$$f(x)=\tan\left(2\pi x+\frac{\pi}{2}\right)$$

$$f'(x)=\sec^{2}\left(2\pi x+\frac{\pi}{2}\right)$$

(g) $f(x)=\dfrac{1}{\left(3x^{4}+x-8\right)^{9}}$

$$f(x)=\frac{1}{(3x^{4}+x-8)^{9}}=(3x^{4}+x-8)^{-9}$$

$$f'(x)=-9\left(12x^{3}+1\right)\left(3x^{4}+x-8\right)^{-10}$$

$$=\frac{-9\left(12x^{3}+1\right)}{\left(3x^{4}+x-8\right)^{10}}$$

(h) $f(x)=e^{x}\left(\log_e x\right)^2$

$$f(x) = e^{x}(\log_e x)^2$$

$$f'(x) = e^{x}\frac{d}{dx}(\log_e x)^2 + (\log_e x)^2 \times \frac{d}{dx}e^{x}$$

$$= \left(e^{x} \times 2 \times \frac{1}{x} \times \log_e x\right) + e^{x}(\log_e x)^2$$

$$= \frac{2e^{x}\log_e x}{x} + e^{x}(\log_e x)^2$$

$$= e^{x}\left(\frac{2\log_e x}{x} + (\log_e x)^2\right)$$

(i) $f(x)=\left(x^4+x\right)^{(2x-1)}$

$$f(x) = (x^4+x)^{2x-1}$$

$$f'(x) = (2x-1)(4x^3+1) \times (x^4+x)^{2x-1-1}$$

$$= (2x-1)(4x^3+1)(x^4+x)^{2x-2}$$

(j) $f(x)=3e^{x}-7\log_e x$

$$f(x) = 3e^{x} - 7\log_e x$$

$$f'(x) = 3e^{x} - \frac{7}{x}$$

(k) $f(x)=(2x-1)^3(x^2-3)^2$

$$f(x) = (2x-1)^3(x^2-3)^2$$

$$f'(x) = (2x-1)^3\frac{d}{dx}(x^2-3)^2 + (x^2-3)^2\frac{d}{dx}(2x-1)^3$$

$$= (2x-1)^3 \times 2(2x)(x^2-3) + (x^2-3)^2 \times 3 \times 2 \times (2x-1)^2$$

$$= 4x(x^2-3)(2x-1)^3 + 6(x^2-3)^2(2x-1)^2$$

$$= (x^2-3)(2x-1)^2\left(4x(2x-1) + 6(x^2-3)\right)$$

$$= (x^2-3)(2x-1)^2\left(8x^2-4x+6x^2-18\right)$$

$$= (x^2-3)(2x-1)^2\left(14x^2-4x-18\right)$$

$$= 2(x^2-3)(2x-1)^2\left(7x^2-2x-9\right)$$

(l) $f(x)=\sqrt{x}\sin x+\cos x$

$$f(x) = \sqrt{x}\sin x + \cos x$$

$$f'(x) = \sqrt{x}\frac{d}{dx}(\sin x) + \sin x\frac{d}{dx}(x^{1/2}) + -\sin x$$

$$= \sqrt{x}\cos x + \sin x \times \frac{1}{2} \times x^{-1/2} - \sin x$$

$$= \sqrt{x}\cos x + \frac{\sin x}{2\sqrt{x}} - \sin x$$

(m) $f(x)=\dfrac{2x-\sqrt{x}}{3-x}$

$$f(x) = \frac{2x-\sqrt{x}}{3-x}$$

$$f'(x) = \frac{(3-x)\frac{d}{dx}(2x-\sqrt{x}) - (2x-\sqrt{x})\frac{d}{dx}(3-x)}{(3-x)^2}$$

$$= \frac{(3-x)\left(2-\frac{1}{2}x^{-1/2}\right) + (2x-\sqrt{x})}{(3-x)^2}$$

$$= \frac{(3-x)\left(2-\frac{1}{2\sqrt{x}}\right) + (2x-\sqrt{x})}{(3-x)^2}$$