DERIVATIVES OF EXPONENTIAL FUNCTIONS

If the power on an exponential function (with base e) is a linear expression, the following rules may be applied:

If $y = e^x$ then $\frac{dy}{dx} = e^x$

If $y = ae^{kx+c}$ then $\frac{dy}{dx} = ake^{kx+c}$

To differentiate more complex exponential functions (where the power is not in the form of a linear expression), we apply the Chain Rule.

Note: The power does not change when differentiating exponential expressions.

QUESTION 1

Differentiate the following equations with respect to x:

(a) $y = e^{4x+1}$

(b) $y = -2e^{1-5x}$

Solution

(a) $y = e^{4x+1}$

This equation is in the form of $y = ae^{kx+c}$, where $a = 1$ and $k = 4$.

By Rule: If $y = ae^{kx+c}$ then $\frac{dy}{dx} = ake^{kx+c}$

$\therefore \frac{dy}{dx} = 4e^{4x+1}$

(b) $y = -2e^{1-5x}$

This equation is in the form of $y = ae^{kx+c}$, where $a = -2$ and $k = -5$.

By Rule: If $y = ae^{kx+c}$ then $\frac{dy}{dx} = ake^{kx+c}$

$\therefore \frac{dy}{dx} = -2 \times -5e^{1-5x} = 10e^{1-5x}$

DIFFERENTIATING COMPLEX EXPONENTIAL FUNCTIONS

To differentiate more complex trigonometric functions (where the angle is not in the form of a linear expression), we apply the Chain Rule.

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} \quad \textbf{or} \quad f'(x) = f'(u) \times u'$$

QUESTION 2

Find the derivative of $y = -5e^{\left(x^2 - 5x\right)}$.

Solution

Identify the inner function u and write y in terms of u:

Let $y = -5e^u$ where $u = x^2 - 5x$

Find the derivative $\frac{du}{dx}$:

$$\frac{du}{dx} = 2x - 5$$

Find the derivative $\frac{dy}{du}$:

$$\frac{dy}{du} = -5e^u$$

Substitute the derivatives into the Chain Rule:

$$\frac{dy}{dx} = -5e^u \times (2x - 5)$$

Replace u with its original expression and simplify:

$$\frac{dy}{dx} = -5(2x - 5)e^{x^2 - 5x}$$

QUICK CHAIN RULE FOR EXPONENTIAL FUNCTIONS I

$$\frac{dy}{dx} = \text{Derivative of the power} \times \text{Given term}$$

Hint: Given the product/quotient of two exponential functions with the **same base**, write the expression as one term using index laws. Do not apply the Product or Quotient Rule.

Note: Do NOT lower the power on the exponential expression by 1.

QUESTION 3

Differentiate each of the following equations with respect to x.

(a) $y = -5e^{\left(x^2 - 5x\right)}$

Let $y = -5e^u$ where $u = x^2 - 5x$

$$\frac{dy}{dx} = (2x-5) \times -5e^{\left(x^2-5x\right)} = -5(2x-5)e^{\left(x^2-5x\right)}$$

(b) $y = \dfrac{e^{\sin x}}{2}$

$$\frac{dy}{dx} = \cos x \times \frac{e^{\sin x}}{2} = \frac{1}{2}\cos x e^{\sin x}$$

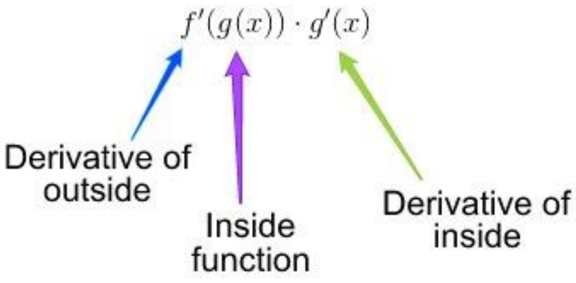
(c) $y = e^{\cos x}.e^x$

$y = e^{\cos(x)+x}$

$$\frac{dy}{dx} = (-\sin x + 1)e^{\cos(x)+x}$$

QUICK CHAIN RULE FOR EXPONENTIAL FUNCTIONS II

Given $f\left(g(x)\right)$:



Step 1: Differentiate the outside function.
Keep the inside function.

Step 2: Multiply by the derivative of the inside function.

QUESTION 3 – REVISITED
Differentiate each of the following equations with respect to x.

(a) $y = -5e^{\left(x^2-5x\right)}$

Step 1: Differentiate the outside function.
Keep the inside function.

Outside function: $y = -5e^{(\ )}$

Derivative: $\dfrac{dy}{dx} = -5e^{(\ )}$

Keep the inside function: $\dfrac{dy}{dx} = -5e^{\left(x^2-5x\right)}$

Step 2: Multiply by the derivative of the inside function.

$$\frac{dy}{dx} = -5e^{\left(x^2-5x\right)} \times (2x-5) = -5(2x-5)e^{\left(x^2-5x\right)}$$

(b) $y = \frac{e^{\sin x}}{2}$

Step 1: Differentiate the outside function.
Keep the inside function.

Outside function: $y = \frac{1}{2}e^{(\ )}$

Derivative: $\frac{dy}{dx} = \frac{1}{2}e^{(\ )}$

Keep the inside function: $\frac{dy}{dx} = \frac{1}{2}e^{(\sin x)}$

Step 2: Multiply by the derivative of the inside function.

$$\frac{dy}{dx} = \frac{1}{2}e^{(\sin x)} \times \cos x = \frac{\cos x e^{(\sin x)}}{2}$$

(c) $y = e^{\cos x}.e^{x}$

$y = e^{\cos(x)+x}$

Step 1: Differentiate the outside function.
Keep the inside function.

Outside function: $y = e^{(\ )}$

Derivative: $\frac{dy}{dx} = e^{(\ )}$

Keep the inside function: $\frac{dy}{dx} = e^{(\cos(x)+x)}$

Step 2: Multiply by the derivative of the inside function.

$$\frac{dy}{dx} = e^{(\cos(x)+x)} \times (-\sin x + 1) = (1 - \sin x)e^{(\cos(x)+x)}$$