THE SCHOOL FOR EXCELLENCE (TSFX)

VCE SPECIALIST MATHEMATICS UNITS 3 & 4

WRITTEN EXAMINATION 2 – 2018

Reading Time: 15 minutes
Writing Time: 2 hours

QUESTION AND ANSWER BOOK

Student Number: **Letter**

Structure of Book

Section	Number of questions	Number of questions to be answered	Number of marks
1	20	20	20
2	6	6	60
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory **DOES NOT** need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are **NOT** permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials Supplied

- Question and answer book of 31 pages
- Formula sheet
- Answer sheet for multiple-choice questions

Instructions

- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are **NOT** permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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QUESTION 4

If $z = 3+4i$ is a complex factor, then the solution set to the equation $z^3 - 3z^2 + 7z + 75$ is

A. $z \in \{-3,\ 3+4i,\ 3-4i\}$

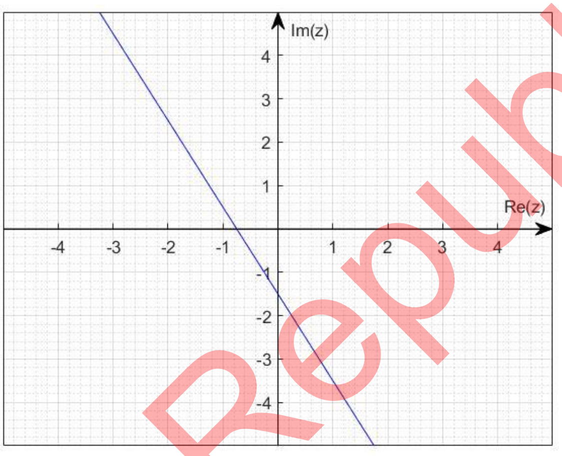
B. $z \in \{-3,\ 3+4i,\ 3+4i\}$

C. $z \in \{3,\ 3+4i,\ 3-4i\}$

D. $z \in \{3+4i,\ 3-4i\}$

E. $z \in \{25,\ 3+4i,\ 3-4i\}$

QUESTION 5



The equation that defines the above straight line in the complex plane is

A. $|z-2| = |z+i|$

B. $Arg(z) = \tan^{-1}(2)$

C. $Arg(z) = \tan^{-1}(2) - 1$

D. $|z+2| = |z-i|$

E. $|z-2| = |z-i|$

QUESTION 6

If $y = \cos^{-1}\left[e^{f(x)}\right]$ and assuming $f(x)$ is differentiable, the value of $\frac{dy}{dx}$ is

A. $\frac{e^{f(x)} f'(x)}{\sqrt{1+\left[e^{f(x)}\right]^2}}$

B. $\frac{e^{f(x)} f'(x)}{\sqrt{1-\left[e^{f(x)}\right]^2}}$

C. $-\frac{e^{f(x)} f'(x)}{\sqrt{1-\left[e^{f(x)}\right]^2}}$

D. $\frac{e^{f(x)} f'(x)}{\sqrt{\left[e^{f(x)}\right]^2 - 1}}$

E. $-\frac{e^{f(x)}}{\sqrt{1-\left[e^{f(x)}\right]^2}}$

QUESTION 7

If $\ln(xy) = 2x$ then $\frac{dy}{dx}$ at $x = 1$ is

A. 0

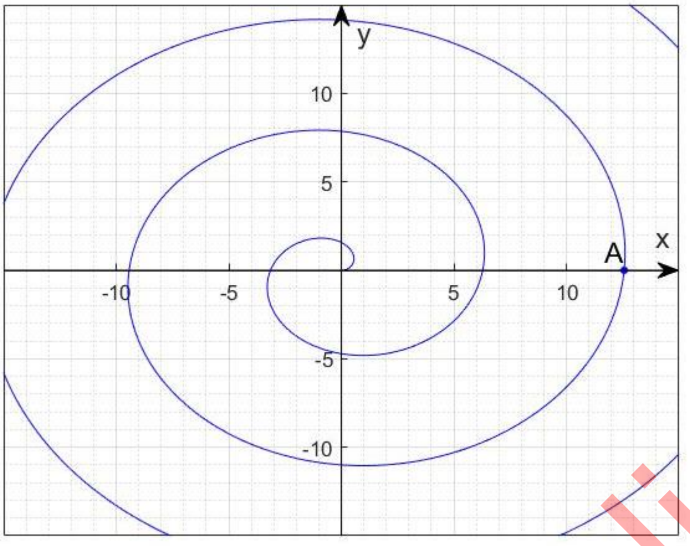
B. e^2

C. $\frac{1}{2}e^2$

D. $1 - \frac{1}{2}e$

E. Undefined

QUESTION 12



The position vector at time $t \geq 0$ for the spiral graph shown above is given by $\underset{\sim}{r}(t) = t\cos t\,\underset{\sim}{i} + t\sin t\,\underset{\sim}{j}$. The instantaneous speed of the particle at the point A is

A. 2

B. 2π

C. 4π

D. $\sqrt{1+4\pi^2}$

E. $\sqrt{1+16\pi^2}$

QUESTION 13

A golf ball is hit from rest horizontally from the ground at a velocity of $70\ m/s$ and lands 200 m down the fairway directly horizontally from where it was hit. If the ball was hit at an angle of θ to the horizontal, then the value of $\sin 2\theta$ is

A. $\frac{1}{5}$

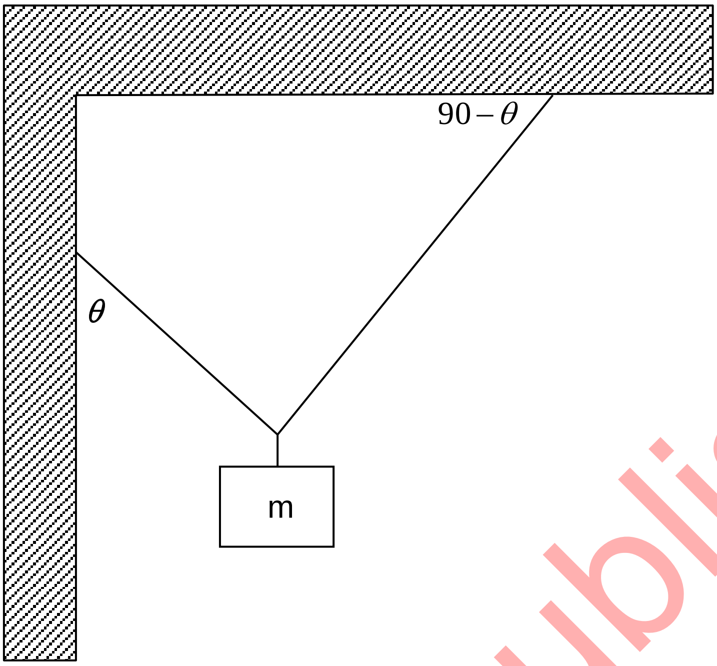
B. $\frac{2}{5}$

C. $\frac{2}{50}$

D. $\frac{4g}{49}$

E. $\frac{g}{49}$

QUESTION 17



A mass m is hanging from a wall and ceiling as shown above. The left string is hanging at an angle of θ to the vertical and the right string is hanging at an angle of $90-\theta$ to the horizontal. The tension in the left string is

A. $\dfrac{mg}{2\cos\theta}$

B. $\dfrac{mg}{2\cos\theta}+1$

C. $\dfrac{mg}{\sin 2\theta}$

D. $\dfrac{mg}{2\sin\theta}$

E. $mg\cos\theta$

QUESTION 18
Cans of soft drink cost \$1.50 each. The expected value and variance of the daily uptake of cash Y if the expected number of cans sold per day is $E(X) = 200$ with variance $Var(X) = 40$ is

A. $E(Y) = 300$ $Var(Y) = 40$

B. $E(Y) = 300$ $Var(Y) = 60$

C. $E(Y) = 300$ $Var(Y) = 90$

D. $E(Y) = 300$ $Var(Y) = 300$

E. $E(Y) = 200$ $Var(Y) = 90$

QUESTION 19
An estimate of the number of eggs collected by a male sea horse with a 90% confidence interval is required. A maximum margin of error of 0.5 is accepted and the population of eggs has a mean of 1000 and a standard deviation of 10. The minimum sample size needed is

A. 32

B. 33

C. 16

D. 1082

E. 1083

QUESTION 20
The mean and standard deviation of the pH of ocean water are 7.25 and 1.1 respectively. 100 samples of ocean water were taken off Antarctica and were found to have an average pH of 7.5. The null hypothesis H_0 is: *The pH of ocean water is equal to 7.25,* while the alternative hypothesis H_1 is: *The pH of ocean water is greater than 7.25.* The $p-$value for this hypothesis test is

A. 0.988

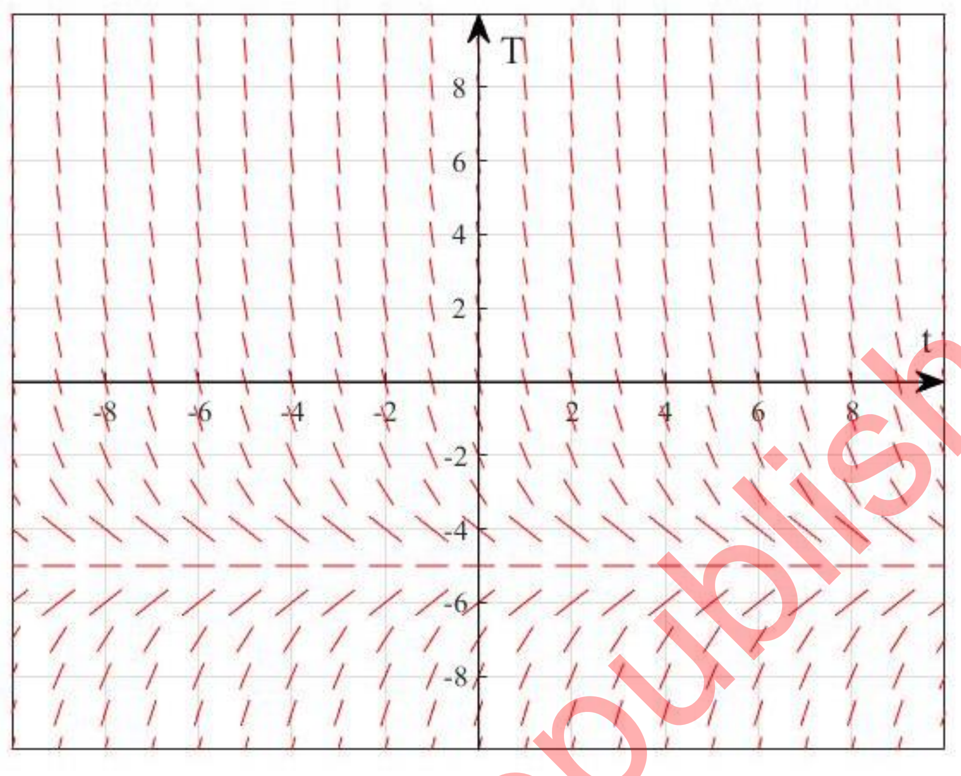
B. 0.846

C. 0.829

D. 0.0115

E. 0.128

QUESTION 3 (12 marks)



a. The differential equation that describes the above slope field is $\frac{dT}{dt} = aT + b$. Write down the signs of the constants a and b and determine how they are related to each other. 2 marks

Newton's law of cooling is described by the differential equation $\frac{dT}{dt} = -k(T - T_a)$ where T is the varying temperature at time t minutes, k is a positive constant and T_a is the constant temperature of the surrounds.

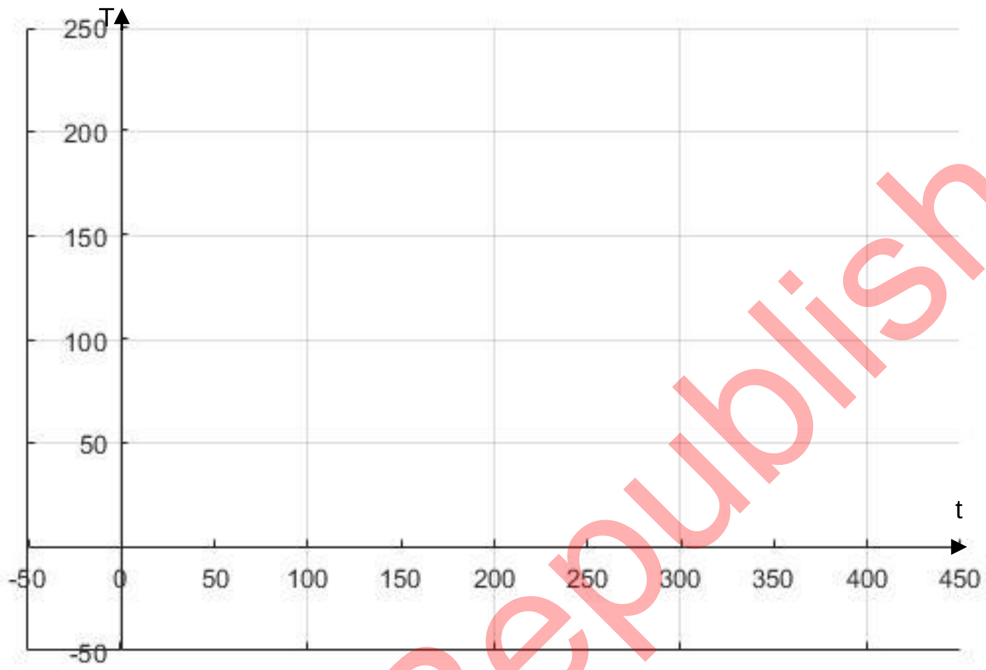
b. Sandra made a cup of soup indoors with a soup temperature of $80°C$ and then took it outdoors to drink. The temperature outdoors is a cold $-5°C$. Find the temperature of the soup as a function of time given these initial conditions. 3 marks

c. **i.** Sandra found that she had to wait 10 minutes before she could drink her soup at a comfortable temperature of $65°C$. Calculate the value of the constant k to four decimal places for the cooling equation. 1 mark

ii. Sandra was getting too cold and left her soup outdoors. How long to the nearest minute does the soup take before it starts to freeze at $0°C$? 1 mark

d. During this time, snow had accumulated in the cup, thus diluting its contents. Sandra then brought it back indoors to heat up in the oven preset at $200°C$. The differential equation that models heating up the soup is now $\frac{dT}{dt} = l(200 - T)$ where $l > 0$. Verify that $T(t) = 200 - Ae^{-lt}$ is a solution to the differential equation and hence evaluate A, assuming that the initial temperature of the soup is still $0°C$. 2 marks

e. Sketch a graph of T in terms of t beginning when Sandra brought the soup outside to leaving it in the oven for a further 30 minutes to warm up to $65°C$. Indicate all of the following coordinates: the starting point, the point ten minutes after, the point when the soup reached $0°C$, the change of graph and the finishing point. Any asymptotes should also be included. 3 marks



QUESTION 6 (10 marks)
The average total rainfall for January in Melbourne is 53.89 mm with a standard deviation of 34.65 mm using rainfall data since 1937. The total January rainfall for the last 15 years is:

35.6	29.8	73.2	45.8	10.8	7	83	100.2	70.2	6.8	28.6	78.6	44.4	34	55.8

a. What is the sample average for the 15 years? 1 mark

b. The local weather service is required to determine whether this random sample provides sufficient evidence that the mean local January rainfall has decreased.
State H_0 and H_1, the null and alternate hypothesis for this situation. 2 marks

c. **i.** Find the p-value correct to four decimal places. 1 mark

ii. Based on a 5% level of significance, state which hypothesis should the weather service accept, giving reasons. 1 mark

iii. What is the whole number value of the minimum sample size required to change the accepted hypothesis? 2 marks

d. What is the probability to three decimal places of Melbourne having a very low total January rainfall of between zero and 55 mm, based on the new sample size calculated above in (c) iii? 1 mark

e. Calculate the approximate 95% confidence interval for the sample mean rainfall with sample size calculated above in (c) iii. Give your answer to 2 decimal places. 2 marks

END OF QUESTION AND ANSWER BOOK