

THE SCHOOL FOR EXCELLENCE (TSFX)

VCE MATHEMATICAL METHODS UNITS 3 & 4

WRITTEN EXAMINATION 2 – 2018

Reading Time: 15 minutes

Writing Time: 2 hours

QUESTION AND ANSWER BOOK

Student
Number:

--	--

--	--	--

--	--	--

Letter

--

Structure of Book

Section	Number of questions	Number of questions to be answered	Number of marks
1	20	20	20
2	4	4	60
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator (memory **DOES NOT** need to be cleared) and, if desired, one scientific calculator. For approved computer-based CAS, their full functionality may be used.
- Students are **NOT** permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials Supplied

- Question and answer book of 22 pages
- Formula sheet
- Answer sheet for multiple-choice questions

Instructions

- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are **NOT** permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION A

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

QUESTION 1

$f : D \rightarrow \mathbb{R}, f(x) = (x - 2)^2 - 1$ has a range of $[3, \infty)$. The domain D could be

- A. $[0, \infty)$
- B. $(0, \infty)$
- C. $(-\infty, 0)$
- D. $(-\infty, 0]$
- E. $[-1, \infty)$

QUESTION 2

$f(x) = (x - 1)(x + 3)P(x) + m(x - 1) + n$, where $P(x)$ is a polynomial and m and n are real numbers.

When $f(x)$ is divided by $(x - 1)$ the remainder is -10 .

When $f(x)$ is divided by $(x + 3)$ the remainder is 2 .

The values of m and n are (respectively)

- A. 2 and -10
- B. 3 and -10
- C. -3 and -10
- D. -10 and 3
- E. 10 and 3

QUESTION 6

The equation $(x+2)^2(x-3)^3 - p = 0$ has only one solution for x when the constant p satisfies the following inequalities

- A. $-108 \leq p \leq 0$
- B. $-108 \leq p < 0$
- C. $p > 1$ and $p < -109$
- D. $p < 0$ and $p > -108$
- E. $p > 0$ and $p < -108$

QUESTION 7

A particular function satisfies the functional equation $f(x+y) = f(x)f(y)$ where $x, y \in \mathbb{R}^+$. A possible rule for the function is

- A. $f(x) = \log_e(x)$
- B. $f(x) = e^{(x-1)}$
- C. $f(x) = e^{-x}$
- D. $f(x) = x^2 - 1$
- E. $f(x) = \cos(-x)$

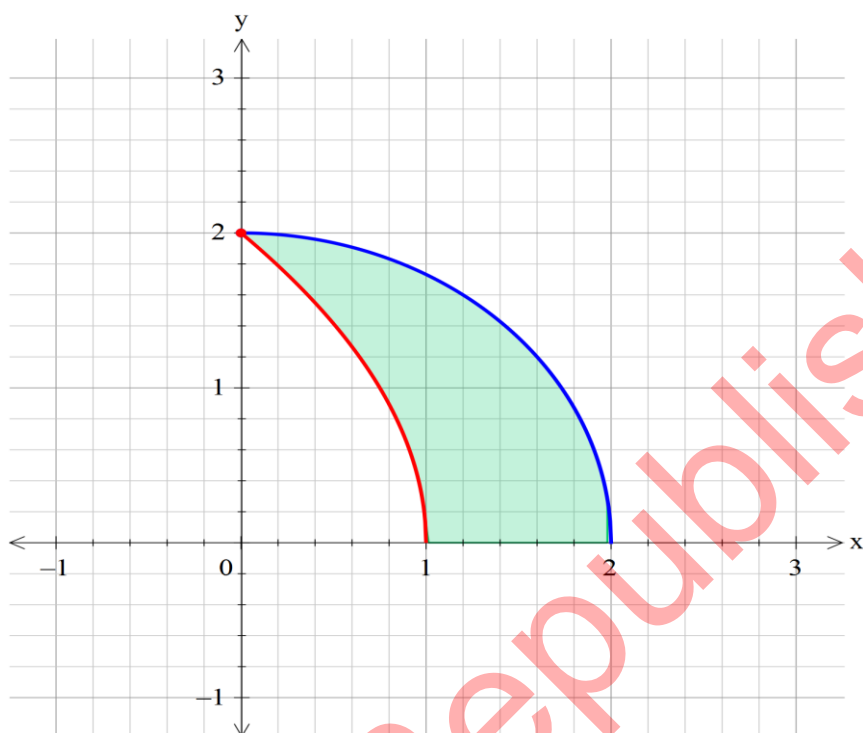
QUESTION 8

If $f(x) = x^3g(x) - 3x$ and $g'(3) = 2$ and $g(3) = -1$ then $f'(3)$ equals

- A. -81
- B. -84
- C. -87
- D. 24
- E. 27

QUESTION 14

The area between the x -axis and the two curves with equations $y = \sqrt{4 - x^2}$ and $y = 2\sqrt{1 - x}$ has been shaded below.



This area (in square units) has a value of

- A. $\frac{3\pi - 4}{3}$
- B. $\frac{10\pi - 13}{10}$
- C. $\frac{3\pi - 2}{3}$
- D. 1.808
- E. $\frac{2\pi - 1}{2}$

QUESTION 17

Westfold Stadium claims that 7 out of every 10 customers gain entry through the turnstile within 6 minutes of queuing at the entrance. Ten random customers have their entry times recorded. Given that at least eight customers gain entry within the six minutes, the probability that there are exactly nine of them who enter within the six minutes, correct to four decimal places, is

- A. 0.1961
- B. 0.1211
- C. 0.3163
- D. 0.3164
- E. 0.3828

QUESTION 18

The random variable, X , has a normal distribution with mean 50 and variance 4. If the random variable, Z , has the standard normal distribution, then the probability that X lies **outside** the interval $[42, 54]$ is

- A. 0.0288
- B. $\Pr(-4 < Z < 2)$
- C. $\Pr(Z > -4) + \Pr(Z < 2)$
- D. $\Pr(Z < -4) + \Pr(Z > 2)$
- E. $\Pr(Z < -2) + \Pr(Z > 1)$

QUESTION 19

15% of oranges in an orchard are 5 cm or smaller in diameter and 30% are 8 cm or more in diameter. If the diameter of oranges is normally distributed, the mean and standard deviation, correct to 4 decimal places are

- A. $\sigma = 1.9221$ and $\mu = 6.9921$
- B. $\sigma = 8.3289$ and $\mu = 3.6323$
- C. $\sigma = 5.8590$ and $\mu = 4.9275$
- D. $\sigma = 3.6323$ and $\mu = 8.3289$
- E. $\sigma = 2.0000$ and $\mu = 6.0000$

SECTION B

Instructions for Section B

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

QUESTION 1 (18 marks)

There are three athletes exercising along a path called **The Sharewalk Path**. This path is shared at any time between runners, walkers and riders.

Sarah, for these purposes called S_1 , is a runner whose running plan follows a Normal distribution where her mean and variance of the time of running her usual distance is 60 minutes and 2 minutes respectively.

- a. Find the probability that Sarah will run her usual distance in less than 58 minutes. Give your answer correct to four decimal places. 1 mark

Sharon, S_2 , is a walker whose exercise plan for a certain number of walks, n , for the probability of finishing that walk, p , follows the Binomial distribution where her mean and variance is 30 and 5 respectively.

- b. (i) Find the values of n and p for Sharon's Distribution. 2 marks

- (ii) Find the probability that Sharon will finish between 30 and 36 walks, correct to four decimal places. 1 mark

- (iii) Find the maximum number of walks that Sharon will complete if the probability of completing at least two walks is at most 99%. 3 marks

Susan, S_3 , is a bike rider whose planned ride on **The Sharewalk Path** is frustrated by the number of times she has to slow down for others sharing the same path. The time for Susan's ride is described by the probability density function

$$S_3(t) = \begin{cases} \frac{1}{10}, & 0 \leq t < 3 \\ a(t-3)^2 + \frac{1}{10}, & 3 \leq t \leq 9 \\ 0, & \text{elsewhere} \end{cases}$$

where t is the time in minutes.

- c. (i) Show that $a = \frac{1}{720}$.

2 marks

- (ii) Determine the expected time, in minutes, for Susan's ride on **The Sharewalk Path**. 1 mark

The records kept by a very large number of runners, walkers and riders of **The Sharewalk Path** indicate that 55% of runners run for more than 60 minutes per day. Sarah is a director of an athletics company and randomly samples 100 such runners.

- d. (i) Find the standard deviation of the sample proportion, \hat{P} . Give your answer correct to four decimal places. 2 marks

- (ii) Find the probability that more than 60% of the sample of 100 runners run for more than 60 minutes per day, correct to four decimal places. Do not use the Normal approximation. 2 marks

- (iii) Using the Normal approximation, find the probability that more than 60% of the sample of 100 runners run for more than 60 minutes per day. Give your answer correct to three decimal places. 2 marks

Similar records kept by a very large number of walkers of **The Sharewalk Path** indicate that 20% of walkers complete their walk. Sharon is a director of the same athletics company as Sarah and randomly samples 200 walkers.

- e. Determine a 95% confidence interval for the population proportion from this sample, correct to two decimal places. 2 marks
