UNIT 3 & 3 MATHEMATICAL METHODS QUIZ

QUESTION 1

The function with rule $f(x)=3\sin\left(2x+\dfrac{\pi}{3}\right)$ will become the function with rule $g(x)=\sin(x)$ through the following ordered sequence of transformations

A Dilation by a factor $\dfrac{1}{2}$ from the *y*-axis; then dilation by a factor 3 from the *x*- axis and finally a translation $\dfrac{\pi}{6}$ to the left, parallel to the *x*-axis.

B Translation of $\dfrac{\pi}{6}$ units to the right parallel to the *x*-axis; then dilation by a factor 2 from the *y*-axis; and finally a dilation by a factor $\dfrac{1}{3}$ from the *x*-axis.

C Dilation by a factor $\dfrac{1}{2}$ from the *y*-axis; then dilation by a factor 3 from the *x*-axis and finally a translation $\dfrac{\pi}{3}$ to the left, parallel to the *x*-axis.

D Translation of $\dfrac{\pi}{3}$ units to the right parallel to the *x*-axis; then dilation by a factor 2 from the *y*-axis; and finally a dilation by a factor $\dfrac{1}{3}$ from the *x*-axis.

E Translation of $\dfrac{\pi}{6}$ units to the left parallel to the *x*-axis; then dilation by a factor 2 from the *y*-axis; and finally a dilation by a factor $\dfrac{1}{3}$ from the *x*-axis.

QUESTION 2

If $\log_e x=\log_e(x-1)+b$ then x is equal to

A. $\dfrac{e^b}{1-e^b}$

B. $\dfrac{1}{e^b-1}$

C. $\log_e\dfrac{x}{x-1}$

D. $\dfrac{1}{1-e^b}$

E. $\dfrac{e^b}{e^b-1}$

QUESTION 3

Let $f(x) = a\sin(2x)$ and $g(x) = b$, where $0 < x < \frac{3\pi}{2}$ and *a* and *b* are positive integers.

Which of the following statements is **not** true?

A If $b > a$ there are no real solutions to the equation $f(x) = g(x)$.

B If $0 < b < a$ there are 4 solutions to the equation $f(x) = g(x)$.

C If $0 < b < a$ there are 4 solutions to the equation $f(x) = -g(x)$.

D If $a = b$ there is 1 solution to the equation $f(x) = -g(x)$.

E If $a = b$ there are 2 solutions to the equation $f(x) = g(x)$.

QUESTION 4

$\sin\theta = \sin\left(\frac{5\pi}{7}\right)$, $\theta \in R$ is equal to

A $\theta = \pi k + (-1)^k \frac{5\pi}{7}$

B $\theta = \pi k + (-1)^k \frac{2\pi}{7}$

C $\theta = 2\pi k + (-1)^k \frac{2\pi}{7}$

D $\theta = 2\pi k + (-1)^k \frac{5\pi}{7}$

E $\theta = 2\pi k - (-1)^k \frac{5\pi}{7}$

QUESTION 5

The function with rule $f(x) = \begin{cases} (x-a)^3 + 2, & x \le 0 \\ bx + \cos x, & x > 0 \end{cases}$ is differentiable for all values of x if

A $a = 0$ and $b = 1$

B $a = 1$ and $b = -3$

C $a = 1$ and $b = 3$

D $a = -1$ and $b = -3$

E $a = -1$ and $b = 3$

QUESTION 6

If $f(x)=2^x$ which of the following functions produces the greatest value for $f\left(g(x)\right)$ for all $x>1$ where $c>1$?

A $g(x)=cx$

B $g(x)=x-c$

C $g(x)=\dfrac{c}{x}$

D $g(x)=0$

E $g(x)=1$

QUESTION 7

The function f satisfies the functional equation $f(x-y)=\dfrac{f(x)}{f(y)}$ where x and y are any non-zero real numbers. A possible rule for the function f is

A $f(x)=ax$

B $f(x)=a\log_e x$

C $f(x)=2^{ax}$

D $f(x)=f(x-y)+yf(x)$

E $f(x)=1-x^2$

QUESTION 8

If $f'(x)=-f(x)$ and $f(1)=1$, then $f(x)=$

A $\dfrac{1}{2}e^{-2x+2}$

B e^{-x-1}

C e^{1-x}

D e^{-x}

E $-e^x$

QUESTION 9

If $y = \tan u$, $u = v - \frac{1}{v}$ and $v = \log_e x$, the value of $\frac{dy}{dx}$ at $x = e$ is:

A 0

B $\frac{1}{e}$

C 1

D $\frac{2}{e}$

E $\sec^2(e)$

QUESTION 10

Let f be a one-to-one differentiable function such that $f(3) = 7,\ f(7) = 8,\ f'(3) = 2$ and $f'(7) = 3$. The function g is differentiable and $g(x) = f^{-1}(x)$ for all x. $g'(7)$ is equal to

A $\frac{1}{2}$

B 2

C $\frac{1}{6}$

D $\frac{1}{8}$

E $\frac{1}{3}$

QUESTION 11

If $y=\begin{cases}\cos x & for \ \left\{x:0\le x\le \frac{\pi}{2}\right\}\cup\left\{x:\frac{3\pi}{2}\le x\le 2\pi\right\} \\ -\cos x & for \ \left\{x:\frac{\pi}{2}<x<\frac{3\pi}{2}\right\}\end{cases}$,

the rate of change of y with respect to x at $x=k, \frac{\pi}{2}<k<\frac{3\pi}{2}$, is:

A $-\sin(k)$

B $\sin(k)$

C $-\cos(k)$

D $-\sin(1)$

E $\sin(1)$

QUESTION 12

Let $g(x)=f(e^{\cos x})$.

If $g'(x)=-\sin xe^{\cos x}\times\left(e^{\cos x}\right)^3$ find $f(x)$.

Solution

QUESTION 13

If $\int_{1}^{2} f(x-c)\,dx = 5$ where c is a constant, then $\int_{1-c}^{2-c} f(x)\,dx =$

A $5+c$

B 5

C $5-c$

D $c-5$

E -5

QUESTION 14

A region in the plane is bounded by the graph of $y=\frac{1}{x}$, the x axis, the line $x=m$, and the line $x=2m,\ m>0$. The area of this region

A is independent of m.

B increases as m increases.

C decreases as m increases.

D decreases as m increases when $m<\frac{1}{2}$; increases as m increases when $m>\frac{1}{2}$.

E increases as m increases when $m<\frac{1}{2}$; decreases as m increases when $m>\frac{1}{2}$.

QUESTION 15

Which of the following limits is equal to $\int_{3}^{5} x^4\,dx$?

A $\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\left(3+\frac{k}{n}\right)^4\frac{1}{n}$

B $\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\left(3+\frac{k}{n}\right)^4\frac{2}{n}$

C $\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\left(3+\frac{2k}{n}\right)^4\frac{1}{n}$

D $\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\left(3+\frac{2k}{n}\right)^4\frac{2}{n}$

E $\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\left(3+\frac{k}{2n}\right)^4\frac{2}{n}$

ANSWERS

QUESTION 1 Answer is B

QUESTION 2 Answer is E

QUESTION 3 Answer is C

QUESTION 4 Answer is B

QUESTION 5 Answer is C

QUESTION 6 Answer is A

QUESTION 7 Answer is C

QUESTION 8 Answer is C

QUESTION 9 Answer is D

QUESTION 10 Answer is A

QUESTION 11 Answer is B

QUESTION 12 $f(x)=\frac{x^4}{4}+c$

QUESTION 13 Answer is B

QUESTION 14 Answer is A

QUESTION 15 Answer is D