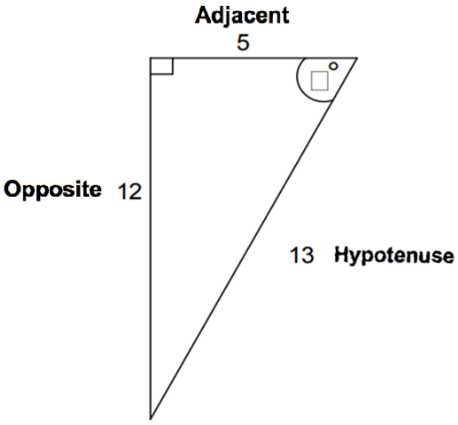
TRIGONOMETRY

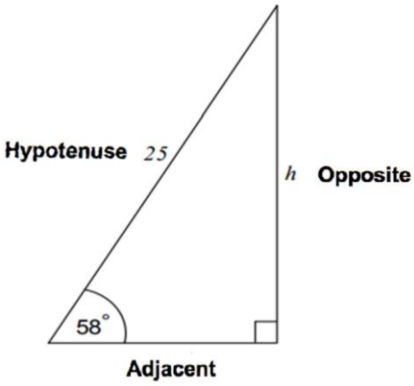
SECTION 1: TRIGONOMETRY BASED ON RIGHT ANGLED TRIANGLES

QUESTION 1

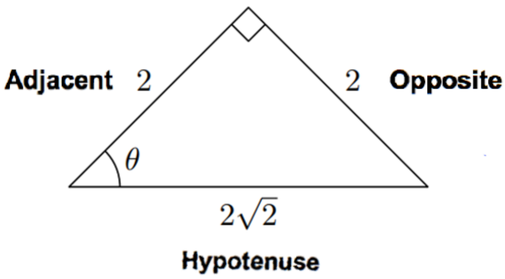
(a)



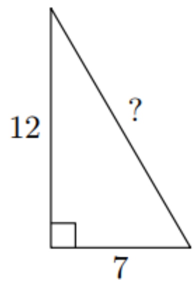
(b)



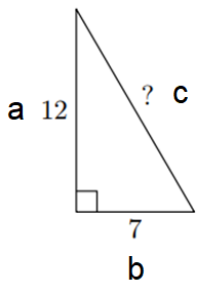
(c)



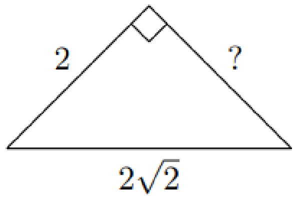
QUESTION 2
Find the unknown length in the following triangle.



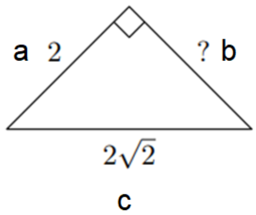
Solution

Step #	**Instruction**	**Your Workings**
Step 1:	Label the triangle in terms of a, b and c where c represents the longest side or hypotenuse.	
Step 2:	Substitute values into $c^2 = a^2 + b^2$ and solve for the required value.	$c^2 = a^2 + b^2$ $c^2 = 12^2 + 7^2$ $c^2 = 193$ $c = \sqrt{193}$

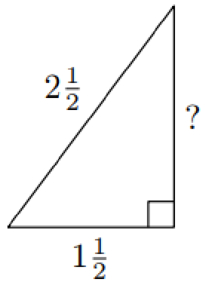
QUESTION 3
Find the unknown length in the following triangle.



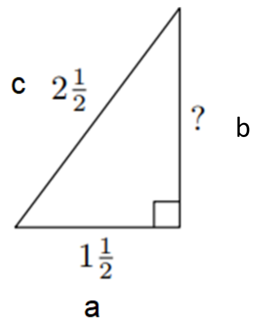
Solution

Step #	Instruction	Your Workings
Step 1:	Label the triangle in terms of a, b and c where c represents the longest side or hypotenuse.	
Step 2:	Substitute values into $c^2 = a^2 + b^2$ and solve for the required value.	$c^2 = a^2 + b^2$ $(2\sqrt{2})^2 = 2^2 + b^2$ $8 = 4 + b^2$ $b^2 = 4$ $b = \sqrt{4} = 2$

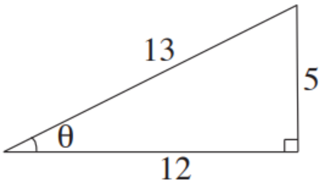
QUESTION 4
Find the unknown length in the following triangle.



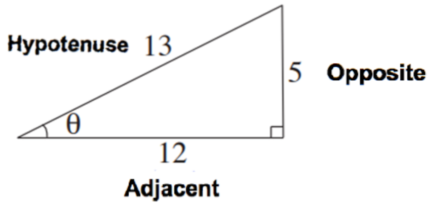
Solution

Step #	**Instruction**	**Your Workings**
Step 1:	Label the triangle in terms of a, b and c where c represents the longest side or hypotenuse.	
Step 2:	Substitute values into $c^2 = a^2 + b^2$ and solve for the required value.	$c^2 = a^2 + b^2$ $(2.5)^2 = (1.5)^2 + b^2$ $6.25 = 2.25 + b^2$ $b^2 = 4$ $b = \sqrt{4} = 2$

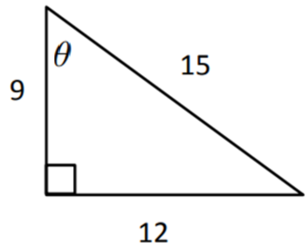
QUESTION 5
Find sin, cos and tan of the angle marked.



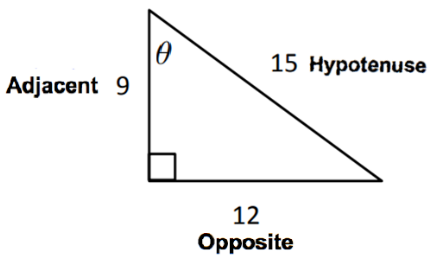
Solution

Step #	**Instruction**	**Your Workings**
Step 1:	Label each side of the triangle with its name.	
Step 2:	Substitute known values into SOHCAHTOA.	$\sin\theta = \frac{O}{H} = \frac{5}{13}$ $\cos\theta = \frac{A}{H} = \frac{12}{13}$ $\tan\theta = \frac{O}{A} = \frac{5}{12}$

QUESTION 6
Find sin, cos and tan of the angle marked.

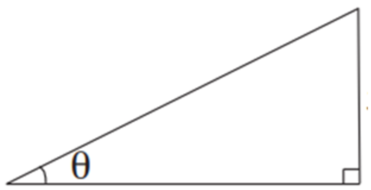


Solution

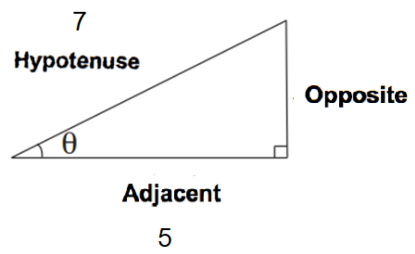
Step #	**Instruction**	**Your Workings**
Step 1:	Label each side of the triangle with its name.	
Step 2:	Substitute known values into SOHCAHTOA.	$\sin\theta = \frac{O}{H} = \frac{12}{15}$ $\cos\theta = \frac{A}{H} = \frac{9}{15}$ $\tan\theta = \frac{O}{A} = \frac{12}{9} = \frac{4}{3}$

QUESTION 7

In the following diagram, $\cos\theta = \frac{5}{7}$. What is the value of $\sin\theta$?

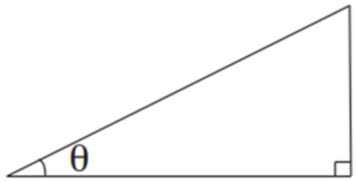


Solution

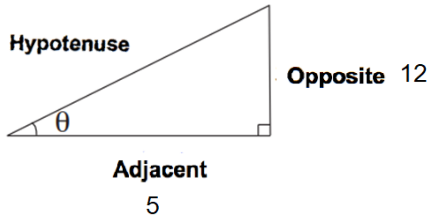
Step #	**Instruction**	**Your Workings**
Step 1:	Label each side of the triangle with its name and known value.	$\cos\theta = \frac{5}{7} = \frac{ADJ}{HYP}$ 
Step 2:	Use Pythagoras' Theorem to find the length of the third side.	$c^2 = a^2 + b^2$ $7^2 = 5^2 + b^2$ $b^2 = 49 - 25$ $b = \sqrt{24} = \sqrt{4\times 6} = 2\sqrt{6}$
Step 3:	State the rule describing the ratio to be found. Then substitute in known values and state the answer.	SOHCAHTOA $\sin\theta = \frac{O}{H}$ $\sin\theta = \frac{2\sqrt{6}}{7}$

QUESTION 8

In the following diagram, $\tan\theta = \frac{12}{5}$. What is the value of $\cos\theta$?

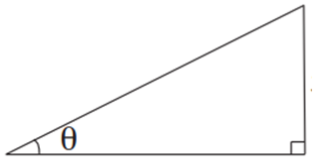


Solution

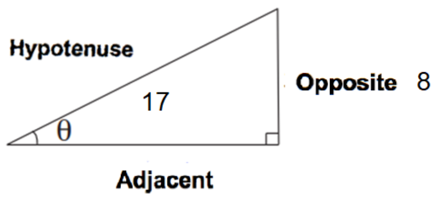
Step #	Instruction	Your Workings
Step 1:	Label each side of the triangle with its name and known value.	$\tan\theta = \frac{12}{5} = \frac{OPP}{ADJ}$ 
Step 2:	Use Pythagoras' Theorem to find the length of the third side.	$c^2 = a^2 + b^2$ $c^2 = 5^2 + 12^2 = 169$ $c = \sqrt{169} = 13$
Step 3:	State the rule describing the ratio to be found. Then substitute in known values and state the answer.	SOH**CAH**TOA $\cos\theta = \frac{A}{H}$ $\cos\theta = \frac{5}{13}$

QUESTION 9

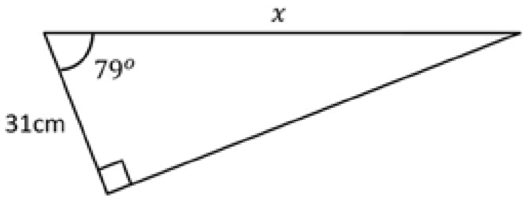
In the following diagram, $\sin\theta = \dfrac{8}{17}$. What is the value of $\cos\theta$ and $\tan\theta$?



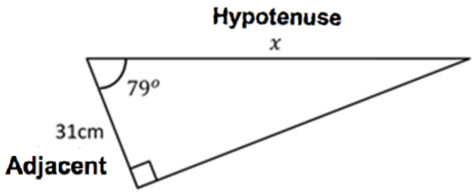
Solution

Step #	Instruction	Your Workings
Step 1:	Label each side of the triangle with its name and known value.	$\sin\theta = \dfrac{8}{17} = \dfrac{OPP}{HYP}$ 
Step 2:	Use Pythagoras' Theorem to find the length of the third side.	$c^2 = a^2 + b^2$ $17^2 = a^2 + 8^2$ $a^2 = 289 - 64 = 225$ $a = \sqrt{225} = 15$
Step 3:	State the rule describing the ratio to be found. Then substitute in known values and state the answer.	SOHCAHTOA $\cos\theta = \dfrac{A}{H}$ and $\tan\theta = \dfrac{O}{A}$ $\cos\theta = \dfrac{15}{17}$ and $\tan\theta = \dfrac{8}{15}$

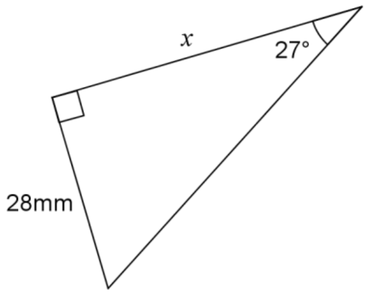
QUESTION 10
Find the length of the unknown side given the following triangle. State your answer to 2 decimal places.



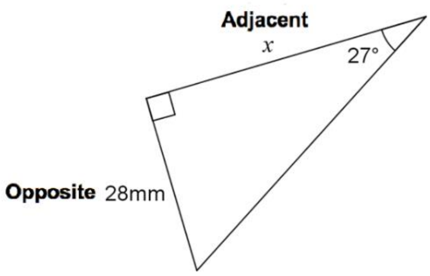
Solution

Step #	**Instruction**	**Your Workings**
Step 1:	Determine whether trigonometric ratios can be used.	Does the triangle have a right angle? Yes. Therefore, we can use SOHCAHTOA.
Step 2:	Label each side of the triangle with its name.	
Step 3:	Identify the ratio that needs to be used. Use the known and unknown lengths.	SOH C**AH** TOA $\cos\theta = \frac{A}{H}$
Step 4:	Substitute in known values into the relevant ratio and solve for the unknown length.	$\cos 79^o = \frac{31}{x}$ $x = \frac{31}{\cos 79^o}$ $x = 162.47\ cm$

QUESTION 11
Find the length of the unknown side given the following triangle. State your answer to 2 decimal places.



Solution

Step #	**Instruction**	**Your Workings**
Step 1:	Determine whether trigonometric ratios can be used.	Does the triangle have a right angle? Yes. Therefore, we can use SOHCAHTOA.
Step 2:	Label each side of the triangle with its name.	
Step 3:	Identify the ratio that needs to be used. Use the known and unknown lengths.	SOH CAH T**OA** $\tan\theta = \frac{O}{A}$
Step 4:	Substitute in known values into the relevant ratio and solve for the unknown length.	$\tan 27^o = \frac{28}{x}$ $x = \frac{28}{\tan 27^o}$ $x = 54.95\ mm$

QUESTION 12
Find the following angles correct to 1 decimal place.

(a) $\sin\theta = 0.5465$
(b) $\cos\theta = 0.707$
(c) $\tan\theta = 1.20$

Solution

(a)

Step #	Instruction	Your Workings
Step 1:	Write the angle in terms of the ratio. $\theta = \sin^{-1}(number)$	$\theta = \sin^{-1}(0.5465)$
Step 2:	Solve for θ using a calculator.	$\theta = 33.1^o$

(b)

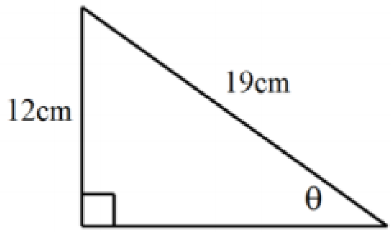
Step #	Instruction	Your Workings
Step 1:	Write the angle in terms of the ratio. $\theta = \sin^{-1}(number)$	$\theta = \cos^{-1}(0.707)$
Step 2:	Solve for θ using a calculator.	$\theta = 45.0^o$

(c)

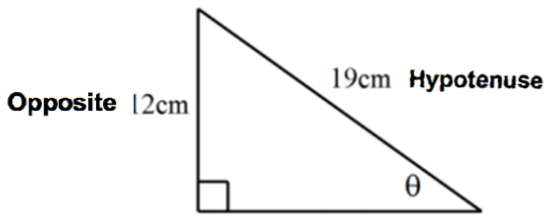
Step #	Instruction	Your Workings
Step 1:	Write the angle in terms of the ratio. $\theta = \sin^{-1}(number)$	$\theta = \tan^{-1}(1.20)$
Step 2:	Solve for θ using a calculator.	$\theta = 50.2^o$

QUESTION 13

Find θ given the following triangle.

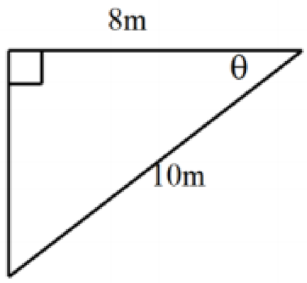


Solution

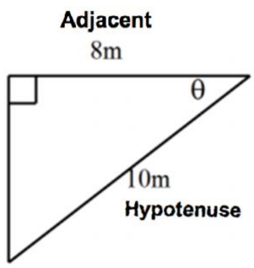
Step #	**Instruction**	**Your Workings**
Step 1:	Determine whether trigonometric ratios can be used.	Does the triangle have a right angle? Yes. Therefore, we can use SOHCAHTOA.
Step 2:	Label the triangle sides with their correct names.	
Step 3:	Identify the ratio that needs to be used. Note that you'll need 2 out of the 3 values in one of the trigonometric ratios.	S**OH** CAH TOA $\sin\theta = \frac{O}{H}$
Step 4:	Calculate the value of θ.	$\sin\theta = \frac{O}{H} = \frac{12}{19}$ $\theta = \sin^{-1}\left(\frac{12}{19}\right) = 39.17^{o}$

QUESTION 14

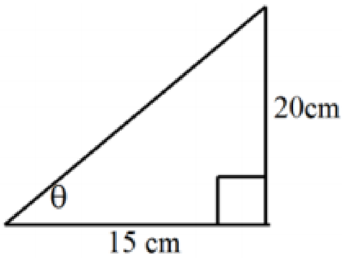
Find θ given the following triangle.



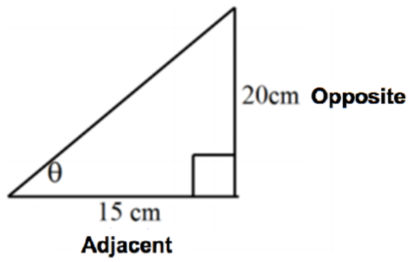
Solution

Step #	**Instruction**	**Your Workings**
Step 1:	Determine whether trigonometric ratios can be used.	Does the triangle have a right angle? Yes. Therefore, we can use SOHCAHTOA.
Step 2:	Label the triangle sides with their correct names.	
Step 3:	Identify the ratio that needs to be used. Note that you'll need 2 out of the 3 values in one of the trigonometric ratios.	SOH C**AH** TOA $\cos\theta = \frac{A}{H}$
Step 4:	Calculate the value of θ.	$\cos\theta = \frac{8}{10} = \frac{4}{5} = 0.8$ $\theta = \cos^{-1}(0.8) = 36.87^o$

QUESTION 15
Find θ given the following triangle.



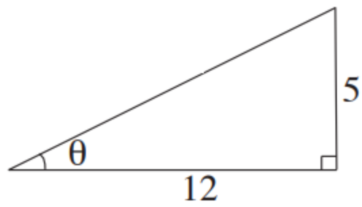
Solution

Step #	**Instruction**	**Your Workings**
Step 1:	Determine whether trigonometric ratios can be used.	Does the triangle have a right angle? Yes. Therefore, we can use SOHCAHTOA.
Step 2:	Label the triangle sides with their correct names.	
Step 3:	Identify the ratio that needs to be used. Note that you'll need 2 out of the 3 values in one of the trigonometric ratios.	SOH CAH T**OA** $\tan\theta = \frac{O}{A}$
Step 4:	Calculate the value of θ.	$\tan\theta = \frac{O}{A} = \frac{20}{15} = \frac{4}{3}$ $\theta = \tan^{-1}\left(\frac{4}{3}\right) = 53.13^o$

QUESTION 16

(a) Find θ given the following triangle.

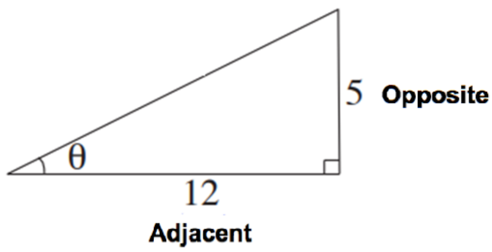
(b) Use trigonometric ratios to find the length of the third side of the triangle.



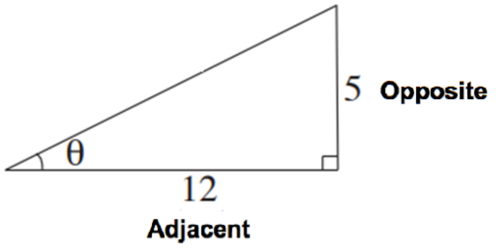
State your answers to 2 decimal places.

Solution

(a)

Step #	Instruction	Your Workings
Step 1:	Determine whether trigonometric ratios can be used.	Does the triangle have a right angle? Yes. Therefore, we can use SOHCAHTOA.
Step 2:	Label the triangle sides with their correct names.	
Step 3:	Identify the ratio that needs to be used. Note that you'll need 2 out of the 3 values in one of the trigonometric ratios.	SOH CAH T**OA** $\tan\theta = \frac{O}{A}$
Step 4:	Calculate the value of θ.	$\tan\theta = \frac{5}{12}$ $\theta = \tan^{-1}\left(\frac{5}{12}\right) = 22.62^{o}$

(b)

Step #	Instruction	Your Workings
Step 1:	Determine whether trigonometric ratios can be used.	Does the triangle have a right angle? Yes. Therefore, we can use SOHCAHTOA.
Step 2:	Label each side of the triangle with its name.	
Step 3:	Identify the ratio that needs to be used. Use the known and unknown lengths.	S**OH** C**AH** TOA $\cos\theta = \frac{A}{H}$ or $\sin\theta = \frac{O}{H}$
Step 4:	Substitute in known values into the relevant ratio and solve for the unknown length.	$\cos 22.62^o = \frac{12}{H}$ $H = \frac{12}{\cos 22.62^o} = 13.00$

QUESTION 17
Do the following triangles have a right angle?

(a) 7, 8, 10
(b) 2, 4.8, 5.2

Solution

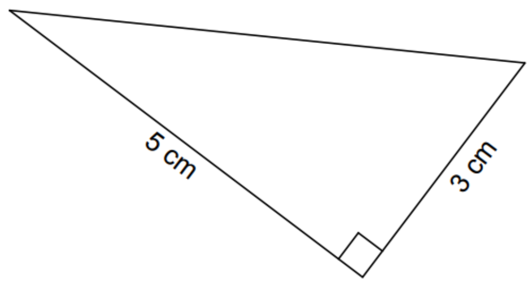
(a)

Step #	**Instruction**	**Your Workings**
Step 1:	Can the given values be obtained by multiplying a common triad by some constant?	No
Step 2:	Does $c^2 = a^2 + b^2$? If YES, the triangle has a right angle. Note that c is always the longest length.	$c^2 = 10^2 = 100$ $a^2 + b^2 = 7^2 + 8^2 = 113$ $c^2 \neq a^2 + b^2$ Therefore, the triangle is not a right-angled triangle.

(b)

Step #	**Instruction**	**Your Workings**
Step 1:	Can the given values be obtained by multiplying a common triad by some constant?	Unsure
Step 2:	Does $c^2 = a^2 + b^2$? If YES, the triangle has a right angle. Note that c is always the longest length.	$c^2 = 5.2^2 = 27.04$ $a^2 + b^2 = 4.8^2 + 2^2 = 27.04$ As $c^2 = a^2 + b^2$, the triangle is a right-angled triangle.

QUESTION 18
Explain why the unknown length in the below triangle is not equal to 4 cm.

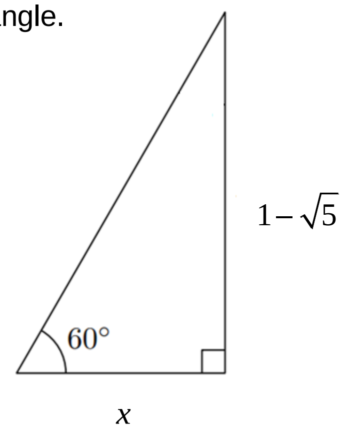


Solution

A known triad is 3, 4, 5. Even though two of the three values are present we cannot automatically assume that the third value is the length of our unknown. For a triad to be valid, the hypotenuse or c must be the longest length, which in this case is 5 cm. As this length belongs to the Opposite or Adjacent side, the triad isn't valid, and $c \neq 4$.

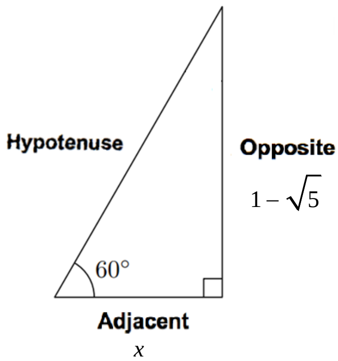
QUESTION 19

Consider the following right-angled triangle.



Show that $x=\dfrac{\sqrt{3}(1-\sqrt{5})}{3}$.

Solution

Step #	**Instruction**	**Your Workings**
Step 1:	Determine whether trigonometric ratios can be used.	Does the triangle have a right angle? Yes. Therefore, we can use SOHCAHTOA.
Step 2:	Label the triangle sides with their correct names.	
Step 3:	Identify the ratio that needs to be used. Note that you'll need 2 out of the 3 values in one of the trigonometric ratios.	SOH CAH T**OA** $\tan\theta=\dfrac{O}{A}$
Step 4:	Substitute values into the ratio and solve.	$\tan 60^o=\dfrac{1-\sqrt{5}}{x}$ $x=\dfrac{1-\sqrt{5}}{\tan 60^o}=\dfrac{1-\sqrt{5}}{\sqrt{3}}=\dfrac{1-\sqrt{5}}{\sqrt{3}}\times\dfrac{\sqrt{3}}{\sqrt{3}}=\dfrac{\sqrt{3}}{3}(1-\sqrt{5})$