TRIGONOMETRY

SECTION 1: TRIGONOMETRY BASED ON RIGHT ANGLED TRIANGLES

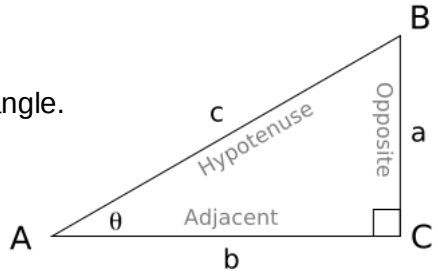
STANDARD NOTATION FOR TRIANGLES

Angle C is usually a right angle (90^o).

Hypothenuse – which is the longest side, opposite the right angle.

Opposite – the side opposite to angle θ.

Adjacent – the side next to angle θ.



PYTHAGORAS' THEOREM

- Pythagoras' Theorem describes the relationships between the lengths of the sides in a right-angled triangle.
- The rule is $c^2 = a^2 + b^2$ where c is the length of the hypotenuse and a and b are the lengths of the other sides in the triangle. Note that it doesn't matter which side is labelled as a or b as long as c is the hypotenuse.
- We use Pythagoras' Theorem when we know the lengths of 2 sides of a triangle, and we want the length of the third side.

TRIGONOMETRIC RATIOS

SOH CAH TOA

$$\sin\theta = \frac{length\ of\ the\ opposite\ side}{length\ of\ the\ hypotenuse} = \frac{opposite}{hypotenuse} = \frac{O}{H}$$

$$\cos\theta = \frac{length\ of\ the\ adjacent\ side}{length\ of\ the\ hypotenuse} = \frac{adjacent}{hypotenuse} = \frac{A}{H}$$

$$\tan\theta = \frac{length\ of\ the\ opposite\ side}{length\ of\ the\ adjacent\ side} = \frac{opposite}{adjacent} = \frac{O}{A}$$

To find the angle θ – We need 2 lengths $\longrightarrow$ Use Trigonometric ratios
To find a length – We need another length and θ $\longrightarrow$ Use Trigonometric ratios
To find a length – We need the other 2 lengths $\longrightarrow$ Use Pythagoras' Theorem

CALCULATING SINE & INVERSE SINE

$$\begin{array}{l}\sin\theta = \text{number} \\ \quad\uparrow \qquad\quad \uparrow \\ \textit{angle} \quad \text{ratio of 2 side } \textit{lengths}\end{array}$$

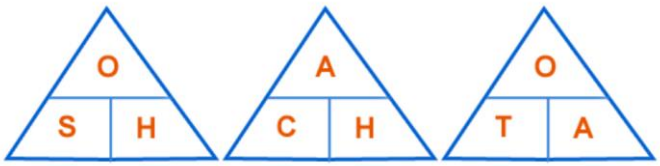
$$\begin{array}{l}\theta = \sin^{-1}(\text{number}) \\ \uparrow \qquad\qquad\quad \uparrow \\ \textit{angle} \qquad \text{ratio of 2 side } \textit{lengths}\end{array}$$

Note: If your angle is in degrees, make sure your calculator is set in DEGREE mode.

FINDING ALL MISSING ANGLES AND SIDES OF A TRIANGLE

Given	Steps
2 sides	Use Pythagoras' Theorem to calculate the length of the third side. Use a trigonometric ratio to find one of the angles. Find the last angle by subtracting known angles from 180^o.
1 side and 1 angle	Use a trigonometric ratio to find the length of one missing side. Use Pythagoras' Theorem to calculate the length of the third side. Find the last angle by subtracting known angles from 180^o.

SOHCAHTOA PYRAMIDS



Simply cover the value you wish to calculate and the remaining values will tell you what mathematical calculations are needed. Note that the horizontal line inside each pyramid represents a division sign (÷) and the vertical line represents a multiplication sign (×).

PYTHAGOREAN TRIPLETS

- Common triads include:

  3, 4, 5
  5, 12, 13
  7, 24, 25
  8, 15, 17 where the longest side is the hypotenuse.

- Other triads can be created by multiplying the above triads by a constant.

  For example: 3, 4, 5 $\xrightarrow{\times 2}$ 6, 8, 10

- If a triad exists, the triangle must be a right-angled triangle and therefore, $c^2 = a^2 + b^2$.

EXACT VALUES OF TRIGONOMETRIC RATIOS

	$0°$	$30°$	$45°$	$60°$	$90°$
$\sin$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

Given other angles, we usually use a calculator to find sines, cosines and tangents.