HOW WELL DO YOU KNOW YOUR COURSE MATERIALS?

These questions (and many others) will be addressed in detail in the TSFX "Unit 2 Exam Revision Lectures" in September 2018.

UNIT 2 MATHEMATICAL METHODS

Question 1
Find the derivative of

(a) $f(x) = -5x^{a+1}.x^{1-3a}$

(b) $y = \dfrac{5}{\sqrt{f(x)}}$

(c) $f\left(\dfrac{1}{(1-x)^3}\right)$

Question 2

If $y = 7x^2 + 5x^3 - \dfrac{3}{2x^2}$, $[-1, \infty)$ find the rate of change of y with respect to x when $x = -1$.

Question 3
Given $h(x) = x^3 g(x) - 3x$ find $h'(3)$ if $g(3) = -1$ and $g'(3) = 2$.

Question 4

A student was asked to prove that the stationary point on the graph of $f(x) = \dfrac{4x}{(x-1)^2}$ is a minimum stationary point. This student found the derivative $f'(x) = \dfrac{-4(x+1)}{(x-1)^3}$, let this equation equal to zero, and then found the corresponding value of x, which is -1. The student then constructed the following table to prove the nature of the stationary point.

x	-2	-1	1
$f'(x)$	$-ve$	0	$+ve$

Why will the student be penalised for using $x = 1$ to test the sign of the derivative?
Even if the student had used the correct values, they would not receive any marks for the above response.
What important set of calculations has the student failed to provide in order to obtain full marks?

Question 5

Find the minimum value of $f(x) = x^3 - 15x^2 + 72x - 100$ for $\{x : 2 \le x \le 7\}$.

Question 6
Let $f'(x) = g'(x) + 3$, $f(0) = 2$ and $g(0) = 1$. Find $f(x)$.

Question 7
Solve for x given that $\sqrt{x+2} = x - 10$, showing all working (no CAS allowed).

Question 8

For what values of p does $x^{\frac{3}{2}} = p + 1$ exist?

Question 9
$f(x) = a\cos x + c$, where a is a positive real number. For what values of x is $f(x) < 0$?

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Question 10
In the following equation, a, b and c are positive constants. The equation $a\sin(x) = c$ is guaranteed to have at least one solution in the interval $0 \le x \le 2\pi$ provided only that:

A $c \le a$

B $c \ge a$

C $b \ge \frac{\pi}{2}$

D $b \le \frac{\pi}{2}$

E $c \le 1$

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ANSWERS

Question 1

(a) $f(x) = -5x^{a+1+1-3a} = -5x^{2-2a}$, $f'(x) = (2-2a) \times -5x^{2-2a-1} = -10(1-a)x^{1-2a}$

(b) $\frac{dy}{dx} = -\frac{1}{2} f'(x) \times 5 f(x)^{-\frac{1}{2}} = \frac{-5f'(x)}{2\sqrt{f(x)}}$

(c) Chain Rule: Let $y = f\left((1-x)^{-3}\right)$ therefore: $u = (1-x)^{-3}$ and $y = f(u)$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = f'\left(\frac{1}{(1-x)^3}\right) \times \frac{3}{(1-x)^4} = \frac{3f'\left(\frac{1}{(1-x)^3}\right)}{(1-x)^4}$$

Question 2
The rate is undefined. You cannot find the derivate at the end point of a domain (i.e. at $x = -1$).

Question 3
Product Rule. $h(x) = x^3 g(x) - 3x$ therefore $h'(x) = x^3 g'(x) + 3x^2 g(x) - 3$
$h'(3) = (3)^3 g'(3) + 3(3)^2 g(3) - 3 = 24$

Question 4
The student be penalised for using $x = 1$ to test the sign of the derivative as the derivative does not exist that that point. To obtain full marks, you must state the numerical value of $f'(x)$ rather than stating that it's positive or negative.

Question 5
Find the minimum occurs at the endpoint at $(2, -8)$

Question 6 $f(x) = g(x) + 3x + 1$.

Question 7
Did you get $x = 7, 14$ as your answer? If yes, then you would have lost the answer mark. Only one of these answers is correct. Could you explain why and be confident of not making similar mistakes in the exams?

PTO

Question 8
The answer is $p \geq -1$.
Could you get this answer in the exams?

Question 9
This question is asking for the conditions underwhich $f(x)$ is below the X axis.
$a\cos x$ is a number that lies between –a and +a (the maximum and minimum values).
To get from $a\cos x$ to $a\cos x + c$, we raise the graph of $a\cos x$ by c units.

$$\therefore Max\,value = a + c$$

For $f(x)$ to be negative, $a + c < 0 \quad \therefore c < -a$

Question 10
Solve this question in the same manner as if numbers were present.

$$a\sin(x+b) = c$$
$$\therefore \sin(x+b) = \frac{c}{a}$$

By definition, $\frac{c}{a}$ must lie between –1 and +1 inclusive i.e. $-1 \leq \frac{c}{a} \leq 1$

$$\therefore \frac{c}{a} \geq -1 \qquad or \qquad \frac{c}{a} \leq 1$$

$$\therefore c \geq -a \qquad\qquad \therefore c \leq a$$

Answer is A