MIXED TECHNIQUES IN DIFFERENTIATION

WORKSHEET 4

Differentiate the following equations with respect to x.

(a) $f(x)=\left(\dfrac{3x-1}{2x+5}\right)^6$

(b) $f(x)=\sqrt[3]{x^2}-\dfrac{1}{\sqrt{x^3}}$

(c) $f(x) = (5x+6)^2 (x-13)^3$

(d) $f(x) = (x^3 - 3x^2 + 11x)^9$

(e) $f(x) = \sqrt{2}x - \frac{1}{\sqrt{x}} + \frac{1}{2}x^6 + 4\pi^2 - \frac{\sqrt{10}}{x^7} + \sqrt[4]{x^5}$

(f) $f(x) = \frac{5x^2 - 7x}{x^2 + 2}$

(g) $f(x) = e^{x^2}.\log_e(\tan x)$

(h) $f(x) = e^x \sin(\cos x)$

(i) $f(x) = \cos\left(\sqrt{x}\right)\sqrt{\cos x}$

(j) $f(x) = \log_e\left(xe^{7x}\right)$

(k) $f(x)=\dfrac{(x-1)^3}{x(x+3)^4}$

SOLUTIONS

(a) $f(x)=\left(\dfrac{3x-1}{2x+5}\right)^6$

$$f(x) = \left(\frac{3x-1}{2x+5}\right)^6$$

$$f'(x) = 6 \times \frac{d}{dx}\left(\frac{3x-1}{2x+5}\right) \times \left(\frac{3x-1}{2x+5}\right)^5$$

$$= 6 \times \left[\frac{(2x+5)3-(3x-1)2}{(2x+5)^2}\right] \times \left(\frac{3x-1}{2x+5}\right)^5$$

$$= 6\left[\frac{6x+15-6x+2}{(2x+5)^2}\right]\left(\frac{3x-1}{2x+5}\right)^5$$

$$= \frac{6(17)(3x-1)^5}{(2x+5)^2(2x+5)^5}$$

$$= \frac{102(3x-1)^5}{(2x+5)^7}$$

(b) $f(x)=\sqrt[3]{x^2}-\dfrac{1}{\sqrt{x^3}}$

$$f(x) = \sqrt[3]{x^2} - \frac{1}{\sqrt{x^3}} = (x^2)^{1/3} - \frac{1}{(x^3)^{1/2}}$$

$$= x^{2/3} - x^{-3/2}$$

$$f'(x) = \frac{2}{3}x^{-1/3} + \frac{3}{2}x^{-5/2}$$

$$= \frac{2}{3x^{1/3}} + \frac{3}{2x^{5/2}}$$

(c) $f(x)=(5x+6)^2(x-13)^3$

$f(x)=(5x+6)^2(x-13)^3$

$f'(x)=(5x+6)^2\frac{d}{dx}(x-13)^3+(x-13)^3\times\frac{d}{dx}(5x+6)^2$

$=(5x+6)^2\times3\times1\times(x-13)^2+(x-13)^3\times2\times5\times(5x+6)^1$

$=3(5x+6)^2(x-13)^2+(x-13)^3\times10\times(5x+6)$

$=3(5x+6)^2(x-13)^2+10(5x+6)(x-13)^3$

$=(x-13)^2\left[3(5x+6)^2+10(5x+6)(x-13)\right]$

$=(x-13)^2\left[3(25x^2+60x+36)+10(5x^2-65x+6x-78)\right]$

$=(x-13)^2\left[75x^2+180x+108+50x^2-590x-780\right]$

$=(x-13)^2(125x^2-410x-672)$

(d) $f(x)=\left(x^3-3x^2+11x\right)^9$

$f(x)=(x^3-3x^2+11x)^9$

$f'(x)=9\times(3x^2-6x+11)(x^3-3x^2+11x)^8$

(e) $f(x)=\sqrt{2}x-\frac{1}{\sqrt{x}}+\frac{1}{2}x^6+4\pi^2-\frac{\sqrt{10}}{x^7}+\sqrt[4]{x^5}$

$f(x)=\sqrt{2}x-\frac{1}{\sqrt{x}}+\frac{1}{2}x^6+4\pi^2-\frac{\sqrt{10}}{x^7}+\sqrt[4]{x^5}$

$=\sqrt{2}x-x^{-1/2}+\frac{1}{2}x^6+4\pi^2-\sqrt{10}x^{-7}+x^{5/4}$

$f'(x)=\sqrt{2}+\frac{1}{2}x^{-3/2}+3x^5+7\sqrt{10}x^{-8}+\frac{5}{4}x^{1/4}$

$=\sqrt{2}+\frac{1}{2x^{3/2}}+3x^5+\frac{7\sqrt{10}}{x^8}+\frac{5x^{1/4}}{4}$

(f) $f(x)=\dfrac{5x^2-7x}{x^2+2}$

$$f(x)=\frac{5x^2-7x}{x^2+2}$$

$$f'(x)=\frac{(x^2+2)(10x-7)-(5x^2-7x)2x}{(x^2+2)^2}$$

$$=\frac{10x^3-7x^2+20x-14-10x^3+14x^2}{(x^2+2)^2}$$

$$=\frac{7x^2+20x-14}{(x^2+2)^2}$$

(g) $f(x)=e^{x^2}.\log_e(\tan x)$

$$f(x)=e^{x^2}.\log_e(\tan x)$$

$$f'(x)=e^{x^2}\frac{d}{dx}\log_e(\tan x)+\log_e(\tan x)\frac{d}{dx}e^{x^2}$$

$$=e^{x^2}\times\frac{\sec^2 x}{\tan x}+2xe^{x^2}\log_e(\tan x)$$

$$=e^{x^2}\left(\frac{\sec^2 x}{\tan x}+2x\log_e(\tan x)\right)$$

(h) $f(x)=e^x\sin(\cos x)$

$$f(x)=e^x\sin(\cos x)$$

$$f'(x)=e^x\times\frac{d}{dx}\sin(\cos x)+\sin(\cos x)\times\frac{d}{dx}e^x$$

$$=e^x\times\cos(\cos x)\times-\sin x+e^x\sin(\cos x)$$

$$=-e^x\sin x\cos(\cos x)+e^x\sin(\cos x)$$

$$=e^x\left(\sin(\cos x)-\sin x\cos(\cos x)\right)$$

(i) $f(x) = \cos\left(\sqrt{x}\right)\sqrt{\cos x}$

$$f(x) = \cos\left(\sqrt{x}\right)\sqrt{\cos x} = \cos\left(x^{1/2}\right) \cdot (\cos x)^{1/2}$$

$$f'(x) = \cos\left(x^{1/2}\right) \times \frac{d}{dx}(\cos x)^{1/2} + (\cos x)^{1/2} \times \frac{d}{dx}\cos\left(x^{1/2}\right)$$

$$= \cos\left(x^{1/2}\right) \times \frac{1}{2} \times -\sin x \times (\cos x)^{-1/2} + (\cos x)^{1/2} \times \frac{1}{2} x^{-1/2} \times -\sin\left(x^{1/2}\right)$$

$$= \frac{-\cos\left(\sqrt{x}\right) \cdot \sin x}{2\sqrt{\cos x}} - \frac{\sin\left(\sqrt{x}\right) \cdot \sqrt{\cos x}}{2\sqrt{x}}$$

(j) $f(x) = \log_e\left(xe^{7x}\right)$

$$f(x) = \log_e(xe^{7x})$$

$$f'(x) = \frac{\frac{d}{dx}(xe^{7x})}{xe^{7x}} = \frac{x \times \frac{d}{dx}(e^{7x}) + e^{7x} \times \frac{d}{dx}(x)}{xe^{7x}}$$

$$= \frac{7xe^{7x} + e^{7x}}{xe^{7x}}$$

$$= \frac{e^{7x}(7x+1)}{xe^{7x}}$$

$$= \frac{7x+1}{x}$$

(k) $f(x)=\dfrac{(x-1)^3}{x(x+3)^4}$

$$f(x)=\frac{(x-1)^3}{x(x+3)^4}$$

$$f'(x)=\frac{x(x+3)^4\cdot\frac{d}{dx}(x-1)^3-(x-1)^3\cdot\frac{d}{dx}x(x+3)^4}{(x(x+3)^4)^2}$$

$$=\frac{x(x+3)^4\cdot 3(x-1)^2-(x-1)^3\left(x\cdot 4(x+3)^3+(x+3)^4\right)}{x^2(x+3)^8}$$

$$=\frac{3x(x+3)^4(x-1)^2-(x-1)^3(x+3)^3(4x+x+3)}{x^2(x+3)^8}$$

$$=\frac{(x+3)^3(x-1)^2\left(3x(x+3)-(x-1)(5x+3)\right)}{x^2(x+3)^8}$$

$$=\frac{(x-1)^2\left(3x^2+9x-(5x^2+3x-5x-3)\right)}{x^2(x+3)^5}$$

$$=\frac{(x-1)^2(3x^2+9x-5x^2+2x+3)}{x^2(x+3)^5}$$

$$=\frac{(x-1)^2(3+11x-2x^2)}{x^2(x+3)^5}$$