MODELLING WITH QUADRATIC FUNCTIONS

WORKSHEET 1

QUESTION 1

John and Sarah are each saving money for a car. The total amount of money John will save is given by the function $f(x) = 60 + 5x$. The total amount of money Sarah will save is given by the function $g(x) = x^2 + 46$. After how many weeks, x, will they have the same amount of money saved?

Solution

QUESTION 2

Mr. Kiefer's Physics class investigates what happens when a ball is given an initial push, rolls up, and then back down an inclined plane. The class finds that the equation $y=-x^2 + 6x$ accurately predicts the ball's position, y, after rolling x seconds.

(a) When is the ball on the ground?
(b) How long was the ball rolling up the inclined plane?
(c) What is the vertex of this function?
(d) In terms of position and time, what does the vertex represent?

Solution

QUESTION 3
A hotel entrance makes a parabolic arch that be represented by the quadratic function $y = -x^2 - 4x + 12$ where y is the height of the arch and x is the distance from wall to wall in metres. What is the greatest distance between the two walls of the arch?

Solution

QUESTION 4

A beekeeper's hives are making honey at a constant rate; yet the price of honey is going steadily down. Let t = the time in days from the start of the honey season. The profit from honey sales as a function of time is given by: $P(t) = -17t^2 + 2040t + 121$

(a) Sketch the graph the function labelling key features. State decimal values correct to 2 places.

(b) State the domain and range.

(c) After how many days should he harvest his honey in order to realise a maximum profit?

Solution

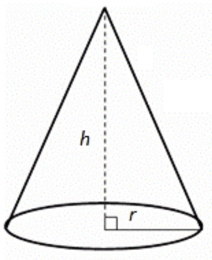
QUESTION 5

The ratio of the base radius and height of a right circular cone is 4:3. If the cost of enameling the curved surface of the cone at 4 cents per $1\,cm^3$ is \$502.52, find the volume of liquid that could fit into the cone in litres. State your answer correct to 2 decimal places.

$$1000\,cm^3 = 1L$$

$$V = \frac{\pi r^2 h}{3}$$

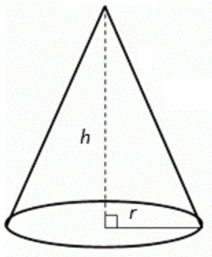
$$SA = \pi r^2 + \pi r\sqrt{h^2 + r^2}$$



Solution

QUESTION 6

The cost of painting the total surface area of a cone at 3 cents per cm^2 is $190.08. Find the volume of the cone if its slant height is 75cm. State your answer correct to the nearest integer.



$1000\,cm^3 = 1L$

$V = \frac{\pi r^2 h}{3}$

$SA = \pi r\left(r + \sqrt{h^2 + r^2}\right)$

Solution

QUESTION 7
Susan throws a softball upward into the air at a speed of 32 feet per second from a 40-foot platform. The height of the ball after t seconds is given the function $d(t) = -16t^2 + 32t + 40$.

(a) What is the maximum height of the softball?
(b) How many seconds does it take to reach the ground after first being thrown upward?

Solution

QUESTION 8
A radiation control point is set up near a solid waste disposal facility. The pad on which the facility is set up measures 20 meters by 30 meters. If the health physicist sets up controlled walkway around the pad that reduces the area by 264 square meters, how wide is the walkway?

Solution

QUESTION 9
A student club is planning a fundraising car wash. Last year they charged $10 per vehicle and washed 120 vehicles. They would like to earn more money this year. For every $1 increase in price, they know they will wash 5 fewer vehicles.

(a) Write the quadratic function to model this situation.
(b) Determine the best price to charge for the car wash and the revenue expected at that price.

Solution

QUESTION 10
An open-top rectangular storage container must have a volume of 10 cubic metres. In addition, the length of the base must be twice the width of the base. If the cost of the base is $10 per square metre and the cost of each side is $6 per square metre:

(a) Show that $h = \frac{5}{w^2}$.

(b) Show that $\text{Cost} = 20w^2 + \frac{180}{w}$.

(c) Find the minimum cost of a container. State your answer correct to the nearest cent.

Solution

SOLUTIONS

QUESTION 1

Let $x^2 + 46 = 60 + 5x$

$x^2 - 5x - 14 = 0$

$(x-7)(x+2) = 0$

$x = -2, 7$

As time can't be negative, $x = 7$ weeks

QUESTION 2

(a)

Find x when $y = 0$

$y = -x^2 + 6x = -x(x-6) = 0$

$\therefore x = 0, 6$

At 0 and 6 seconds

(b) 3 seconds

(c)

$y(3) = -(3)^2 + 6(3) = -9 + 18 = 9$

Vertex is $(3, 9)$

(d) The point where the balls stops and changes its direction of motion.

QUESTION 3

Find the distance between the X intercepts

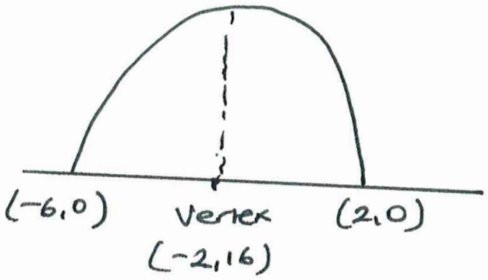
$y = -x^2 - 4x + 12$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{4 \pm \sqrt{(-4)^2 - 4(-1)(12)}}{2(-1)} = \frac{4 \pm \sqrt{16+48}}{-2}$

$= \frac{4 \pm \sqrt{64}}{-2}$

$= \frac{4 \pm 8}{-2}$

$= -6, 2$



Distance between X intercepts = Greatest distance between walls = 8m

QUESTION 4

(a)

$P(t) = -17t^2 + 2040t + 121$

Xint, Let $y = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-2040 \pm \sqrt{(-17)^2 - 4(-17)(121)}}{(2 \times -17)}$

$x = -0.059284,\ 120.059$

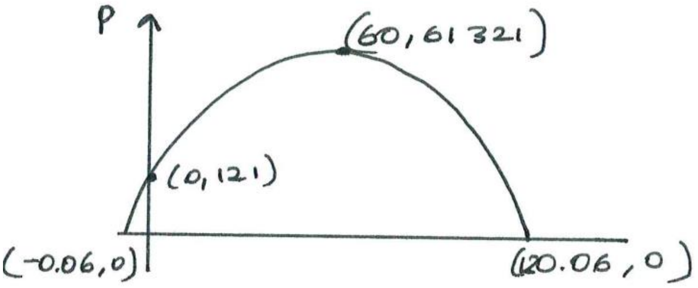
Yint, Let $x = 0$

$y = 121 \quad \therefore (0, 121)$

Vertex

$x\text{coord} = \frac{-b}{2a} = \frac{-2090}{-39} = 60$

$\therefore (60, 61321)$



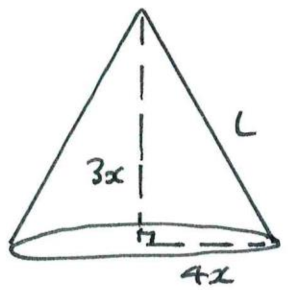
(b)

Domain : $[0, 120.06]$

Range: $[0, 61321]$

(c) 60 days

QUESTION 5



$L^2 = (3x)^2 + (4x)^2 = 25x^2$

$\therefore L = 5x$

Curved surface area $= \dfrac{50252}{4} = 12{,}563\ cm^2$

$= \pi r\sqrt{h^2+r^2} = \pi \times 4x \times 5x$

$= 20\pi x^2$

Equating gives: $20\pi x^2 = 12{,}563$

$x^2 = 199.87$

$x = \pm 14.14cm$

As lengths can't be negative, $x = 14.14$ cm.

$V = \pi r^2 \dfrac{h}{3} = \pi (4x)^2 \left(\dfrac{3x}{3}\right)$

$= \pi 16 x^2 \times x$

$= 16\pi x^3$

$= 16\pi (14.14)x^3$

$= 142{,}108\ cm^3$

$= 142.11\ L$

QUESTION 6

$TSA = \frac{19,008}{3} = 6336 \text{ cm}^2$

TSA of right circular cone $= \pi r\left(r + \sqrt{h^2 + r^2}\right)$

$= \pi r(r + 75)$

Equating gives: $\pi r(r + 75) = 6336$

$r(r+75) = \frac{6336}{\pi}$

$r^2 + 75r = 2016$

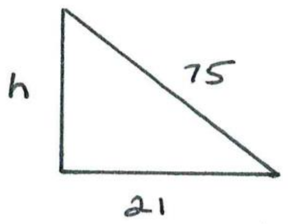
$r^2 + 75r - 2016 = 0$

$(r+96)(r-21) = 0$

$r = -96 \text{ or } 21$

As the radius can't be negative, $r = 21\text{cm}$

Find h:



$75^2 = h^2 + 21^2$

$\therefore h^2 = 75^2 - 21^2$

$h^2 = 5184$

$h = 72$

Find V:

$V = \frac{\pi r^2 h}{3} = \frac{\pi(21)^2(72)}{3} = 33,250.6$

$= 33,251 \text{ cm}^3$

QUESTION 7

(a)

$d(t) = -16t^2 + 32t + 40$

x coordinate of the turning point $= -\frac{b}{2a}$

$= \frac{-32}{2(-16)}$

$= 1$

$\therefore$ Maximum height occurs when $t = 1$ sec

$d(1) = -16 + 32 + 40 = 56$ feet

(b)

Find t when $d = 0$

$-16t^2 + 32t + 40 = 0$

$-8(2t^2 - 4t - 10) = 0$

$2t^2 - 4t - 10 = 0$

$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{4 \pm \sqrt{16 - 4(2)(-5)}}{2(2)} = \frac{4 \pm \sqrt{16 + 40}}{4}$

$= \frac{4 \pm \sqrt{56}}{4}$

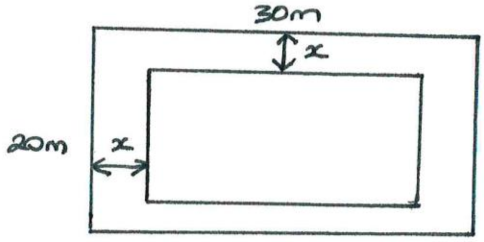
$= \frac{4 \pm \sqrt{4 \times 14}}{4}$

$= \frac{2 \pm \sqrt{14}}{2}$

As time can't be negative, $t = \frac{2 + \sqrt{14}}{2}$ sec

QUESTION 8

Let $x =$ width of walkway



Length of pad without walkway $= 30 - 2x$

width of pad without walkway $= 20 - 2x$

Area of pad without walkway:

$A = L \times W$

$A = (30 - 2x)(20 - 2x) = (30 \times 20) - 264$

$(30 - 2x)(20 - 2x) = 336$

$4x^2 - 100x + 264 = 0$

$x^2 - 25x + 66 = 0$

$(x - 22)(x - 3) = 0$

$\therefore x = 3, 22$

As $x = 22$ is not a possible dimension, the width of the walkway is 3m.

QUESTION 9

(a)

Revenue = Price × Quantity

Let n = number of unit increases

$$\text{Revenue} = (10+n)(120-5n)$$

$$R = 1200 - 50n + 120n - 5n^2$$

$$R = -5n^2 + 70n + 1200$$

(b)

Find maximum price to charge = y coordinate of turning point

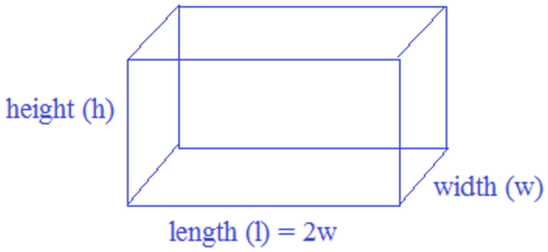
x coordinate of turning point $= \frac{-b}{2a} = \frac{-70}{2(-5)} = 7$

Find R when $n = 7$:

$$R = -5(7)^2 + 70(7) + 1200 = \$1445$$

Price per car $= (10+n) = 10 + 7 = \$17$

QUESTION 10



(a)

$V = 10\ m^3 = l \times w \times h$

$\therefore 2w \times w \times h = 10$

$2w^2h = 10$

$\therefore h = \frac{5}{w^2}$

(b)

Cost = Bottom Cost + Front Cost + Back Cost + Left Cost + Right Cost

= Bottom Cost + 2 × Front Cost + 2 × Right Cost

$= 10(2w^2) + 2(6)(2wh) + 2(6)(wh)$

$= 20w^2 + 24wh + 12wh$

$C = 20w^2 + 36wh$

Substitute $h = \frac{5}{w^2}$ into Cost equation:

$C = 20w^2 + 36w\left(\frac{5}{w^2}\right) = 20w^2 + \frac{180}{w}$

(c)

$C = 20w^2 + 180w^{-1}$

$\frac{dC}{dw} = 40w - 180w^{-2}$

Let $\frac{dC}{dw} = 0$: $40w - \frac{180}{w^2} = 0$

$40w = \frac{180}{w^2}$

$40w^3 = 180$

$w = 1.65$ m

Therefore: width = 1.65 m

length = 3.30 m

height = 1.84 m

Minimum cost = Bottom Cost + 2× Front Cost + 2× Right Cost

$= 10(2 \times 1.65^2) + 2(6 \times 2 \times 1.65 \times 1.84) + 2(6 \times 1.65 \times 1.84)$

$= \$54.45 + \$72.86 + \$36.43$

$= \$163.74$