# UNIT 3 & 4 MATHEMATICAL METHODS
# VCAA EXAMINATION 1 – 2006 TO 2017

## DIFFERENTIATION – RATES OF CHANGE

**QUESTION 1 – 2016 (HARD – HARD)**

Let $f:[-\pi, \pi] \rightarrow R$, where $f(x) = 2\sin(2x) - 1$.

**a.** Calculate the average rate of change of $f$ between $x = -\frac{\pi}{3}$ and $x = \frac{\pi}{6}$. 2 marks

**b.** Calculate the average value of $f$ over the interval $-\frac{\pi}{3} \leq x \leq \frac{\pi}{6}$. 3 marks

# SOLUTIONS

**QUESTION 1**

**a.**

$f(x) = 2\sin(2x) - 1$

$$\text{Av rate change} = \frac{f\left(\frac{\pi}{6}\right) - f\left(-\frac{\pi}{3}\right)}{\frac{\pi}{6} - -\frac{\pi}{3}} \quad (m1)$$

$$= \frac{\sqrt{3} - 1 - \left(-\sqrt{3} - 1\right)}{\pi/2}$$

$$= \frac{4\sqrt{3}}{\pi} \quad (A1)$$

**Difficulty: 32%** of students answered this question correctly
**Average Score:** 1 out of 2

**b.**

$$\text{Av value of } f = \frac{1}{\pi/2} \int_{-\pi/3}^{\pi/6} (2\sin 2x - 1)\, dx \quad (m1)$$

$$= \frac{2}{\pi}\left[-\cos 2x - x\right]_{-\pi/3}^{\pi/6} \quad (m1)$$

$$= \frac{2}{\pi}\left(\left(-\frac{1}{2} - \frac{\pi}{6}\right) - \left(\frac{1}{2} + \frac{\pi}{3}\right)\right)$$

$$= -\frac{2}{\pi}\left(1 + \frac{\pi}{2}\right)$$

$$= -\frac{2}{\pi} - 1 \quad (A1)$$

**Difficulty: 16%** of students answered this question correctly
**Average Score:** 1.3 out of 3