

QUESTION 2 – 2010 (EASY – HARD)

a. Find an antiderivative of $\cos(2x + 1)$ with respect to x .



1 mark

b. Find p given that $\int_2^3 \frac{1}{1-x} dx = \log_e(p)$.

3 marks

QUESTION 3 – 2014 (HARD)

Let $\int_4^5 \frac{2}{2x-1} dx = \log_e(b)$.

2 marks

Find the value of b .

QUESTION 4 – 2015 (HARD)

Evaluate $\int_1^4 \left(\frac{1}{\sqrt{x}} \right) dx$.

SOLUTIONS

QUESTION 1

a.
$$\int \frac{1}{1-2x} dx = -\frac{1}{2} \int \frac{-2}{1-2x} dx \quad (\text{M1})$$
$$= -\frac{1}{2} \log_e |1-2x| \quad +c \text{ not required} \quad (\text{A1})$$

Difficulty: 25% of students answered this question correctly

Average Score: 1.1 out of 2

b.
$$\int_1^4 (\sqrt{x} + 1) dx = \int_1^4 \left(x^{\frac{1}{2}} + 1 \right) dx \quad (\text{M1})$$
$$= \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + x \right]_1^4 \quad (\text{M1})$$
$$= \left(\frac{2}{3} (4)^{\frac{3}{2}} + 4 \right) - \left(\frac{2}{3} (1)^{\frac{3}{2}} + 1 \right)$$
$$= \left(\frac{16}{3} + 4 \right) - \left(\frac{2}{3} + 1 \right) = \frac{23}{3} \quad (\text{A1})$$

Difficulty: 48% of students answered this question correctly

Average Score: 2.2 out of 3

QUESTION 2

a. $\int \cos(2x+1) dx = \frac{1}{2} \sin(2x+1) + c$

Difficulty: 76% of students answered this question correctly
Average Score: 0.8 out of 1

b. $\int_2^3 \frac{1}{1-x} dx = -[\log_e |1-x|]_2^3$ (M1)
 $= -(\log_e |1-3| - \log_e |1-2|)$
 $= -\log_e \left(\frac{2}{1}\right)$
 $= -\log_e 2$

Rearrange to get $-\log_e 2 = \log_e 2^{-1} = \log_e \left(\frac{1}{2}\right)$ (M1)

$p = \frac{1}{2}$ (A1)

Difficulty: 30% of students answered this question correctly
Average Score: 1.5 out of 3

QUESTION 3

$$\int_4^5 \frac{2}{2x-1} dx = \log_e b$$

$$[\log_e (2x-1)]_4^5 = \log_e b \quad (M1)$$

$$\log_e 9 - \log_e 7 = \log_e b$$

$$\log_e \left(\frac{9}{7}\right) = \log_e b$$

$$\therefore b = \frac{9}{7} \quad (A1)$$

Difficulty: 47% of students answered this question correctly
Average Score: 0.6 out of 2

QUESTION 4

$$\begin{aligned}\int_1^4 \left(\frac{1}{\sqrt{x}}\right) dx &= \int_1^4 x^{-1/2} dx \\ &= \left[2x^{1/2}\right]_1^4 \quad (m1) \\ &= 2\sqrt{4} - 2\sqrt{1} \\ &= 2 \quad (A1)\end{aligned}$$

Difficulty: 47% of students answered this question correctly
Average Score: 0.6 out of 2