tsfx

UNIT 3 MATHEMATICAL METHODS

TRIGONOMETRY

EXECUTIVE SUMMARY & EXAM QUESTIONS

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TRIGONOMETRY – REVISION QUESTIONS
TRANSFORMATIONS

Reflection in the Y axis (when n is -ve)

Moves curve

Let angle = 0 and solve for x.
Answer gives you new starting point of curve. i.e. horizontal translation (Translation parallel to θ axis)

Reflection in the θ axis

$$y = -\ a\ trig\ n\ (\theta - \in) + k$$

Moves curve

Translation parallel to Y axis

Amplitude or dilation of factor $|a|$ from the θ axis

Dilation of factor $\left|\frac{1}{n}\right|$ from the Y axis

The value of n affects the period.

For example: $f(x) = 1 - 4\cos\left(2x + \frac{\pi}{2}\right)$ from $g(x) = \cos x$.

- Dilation of factor $\frac{1}{2}$ from the Y axis (or parallel to the X axis).
- Dilation of factor 4 from the X axis (or parallel to the Y axis).
- Reflection in the X axis.
- Translation of $\frac{\pi}{4}$ units in the negative direction along the X axis.
- Translation of 1 units in the positive direction along the Y axis.

QUESTION 1 – EXAM 1

State the transformations required to sketch the graph of $f(x)$ from $g(x)$.

(a) $f(x) = 3\cos\left(2x + \frac{\pi}{4}\right)$, $g(x) = \cos x$.

Dilation from the Y axis **Yes No** **Factor:**	**Dilation from the X axis** **Yes No** **Factor:**
Reflection in the Y axis: **Yes No**	**Reflection in the X axis:** **Yes No**
Translation parallel to the X axis **Yes No** **Number of Units:** **+ve direction -ve direction**	**Translation parallel to the Y axis** **Yes No** **Number of Units:** **+ve direction -ve direction**

(b) $f(x) = 1 - 3\sin(2x - 9)$, $g(x) = \sin x$.

Dilation from the Y axis **Yes No** **Factor:**	**Dilation from the X axis** **Yes No** **Factor:**
Reflection in the Y axis: **Yes No**	**Reflection in the X axis:** **Yes No**
Translation parallel to the X axis **Yes No** **Number of Units:** **+ve direction -ve direction**	**Translation parallel to the Y axis** **Yes No** **Number of Units:** **+ve direction -ve direction**

(c) $f(x) = 2\tan\left(\frac{\pi}{4} - \frac{x}{3}\right) - 5$, $g(x) = \tan x$.

Dilation from the Y axis **Yes No** **Factor:**	**Dilation from the X axis** **Yes No** **Factor:**
Reflection in the Y axis: **Yes No**	**Reflection in the X axis:** **Yes No**
Translation parallel to the X axis **Yes No** **Number of Units:** **+ve direction -ve direction**	**Translation parallel to the Y axis** **Yes No** **Number of Units:** **+ve direction -ve direction**

QUESTION 2 – EXAM 2

The function $f(x)$ is dilated by a factor of 2 from the X axis, and by a factor of 3 from the Y axis, and translated 4 units horizontally in a positive direction from the Y axis and 2 units in a negative direction from the X axis. If $f(x)=\cos(x)$ then the transformed function $g(x)$ would be:

A $2\cos 3(x+4)-2$

B $3\cos 0.5(x+4)-2$

C $2\cos\left(\frac{1}{3}(x+4)\right)-2$

D $2\cos\left(\frac{x-4}{3}\right)-2$

E $3\cos 2(x-4)+2$

Solution

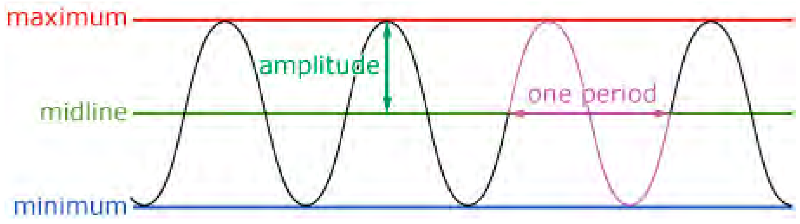
SKETCHING TRIGONOMETRIC FUNCTIONS BY CONSIDERING TRANSFORMATIONS

Step 1: Sketch the base curve i.e. $y = a\sin\theta$, $y = a\cos\theta$ or $y = a\tan\theta$.

Step 2: Take into account any reflections in the θ or Y axes.

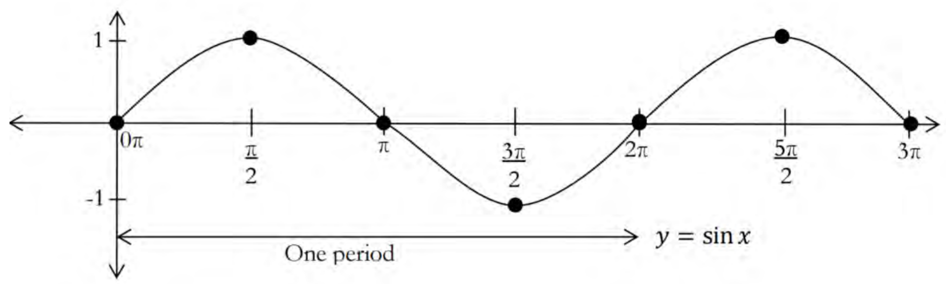
If the sign directly in front of θ is negative, reflect the graph in the Y axis.
If the sign directly in front of a is negative, reflect the graph in the θ axis.

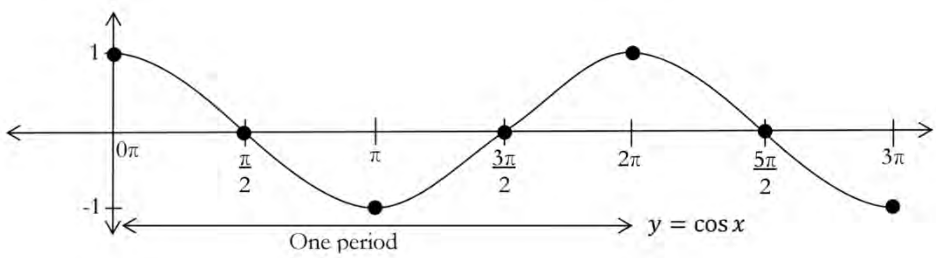
Step 3: Calculate the distance between each key point (the quadrant boundaries) by dividing the period by 4.



The key points on the sine and cosine functions are the θ values (the angles) that correspond to the maximum, minimum, and the points that lie on the horizontal midline.

Note that the midline is the horizontal line that passes exactly in the middle between the graph's maximum and minimum points.





Step 4: Identify the horizontal translation and sketch the graph of $y = atrig\ n(\theta - \in)$.

Let $(\theta - \in) = 0$ and solve for $\in$:

If $\in$ > 0, translate the curve $\in$ units to the right.
If $\in$ < 0, translate the curve $\in$ units to the left.

If the horizontal translation and period are fractions with different denominators, express both fractions with the same denominators before you attempt to sketch the graph.

Step 5: Identify the vertical translation and sketch the graph of $y = atrig\ n(\theta - \in) + k$.

If $k > 0$, translate the curve k units upwards.
If $k < 0$, translate the curve k units downwards.

The order of transformations is critical:

Dilate/Reflect and then Translate (DRT or RDT).

QUESTION 3 – EXAM 1

Sketch $y = 2\cos\left(x + \frac{\pi}{2}\right)$ across $x \in [0, 2\pi]$ showing all transformations.

Solution

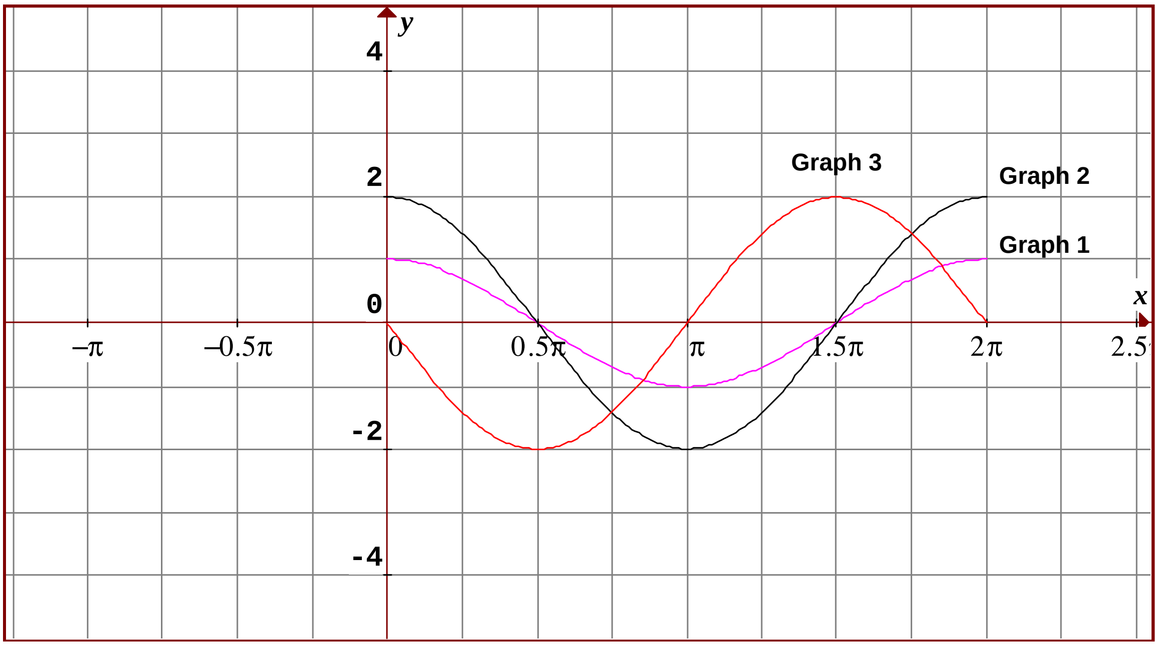
Amplitude: 2

Period: $\frac{2\pi}{1} = 2\pi$

Distance between points: $\frac{Period}{4} = \frac{2\pi}{4} = \frac{\pi}{2}$

Reflections: Nil

Translations: $\frac{\pi}{2} \leftarrow$



Graph 1: Draw $y = \cos(x)$.

Graph 2: Draw $y = 2\cos(x)$ i.e. Stretch curve along Y axis (multiply each value of Y by 2).

Graph 3: Draw $y = 2\cos\left(x + \frac{\pi}{2}\right)$ i.e. Move entire curve $\frac{\pi}{2}$ units to the left.

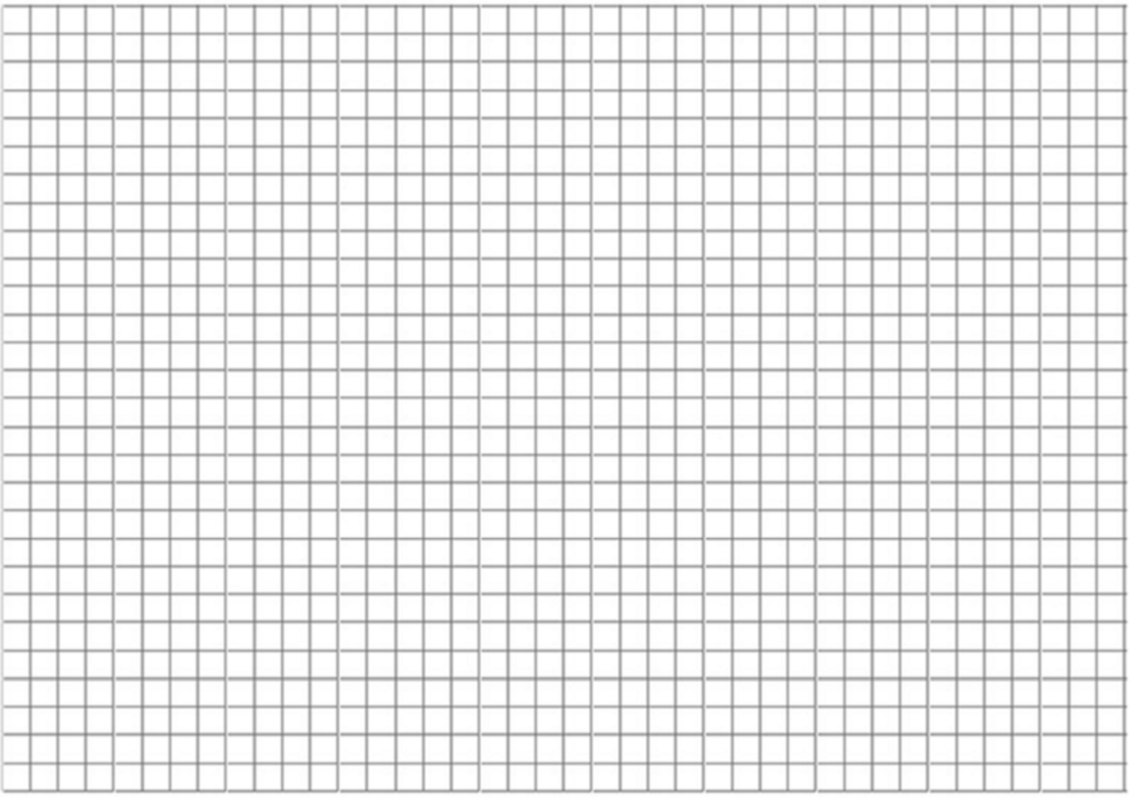
QUESTION 4 – EXAM 1

Sketch the following graphs showing transformations. In each case, show one complete cycle.

(a) $y = -2\cos\left(2x + \frac{\pi}{2}\right)$

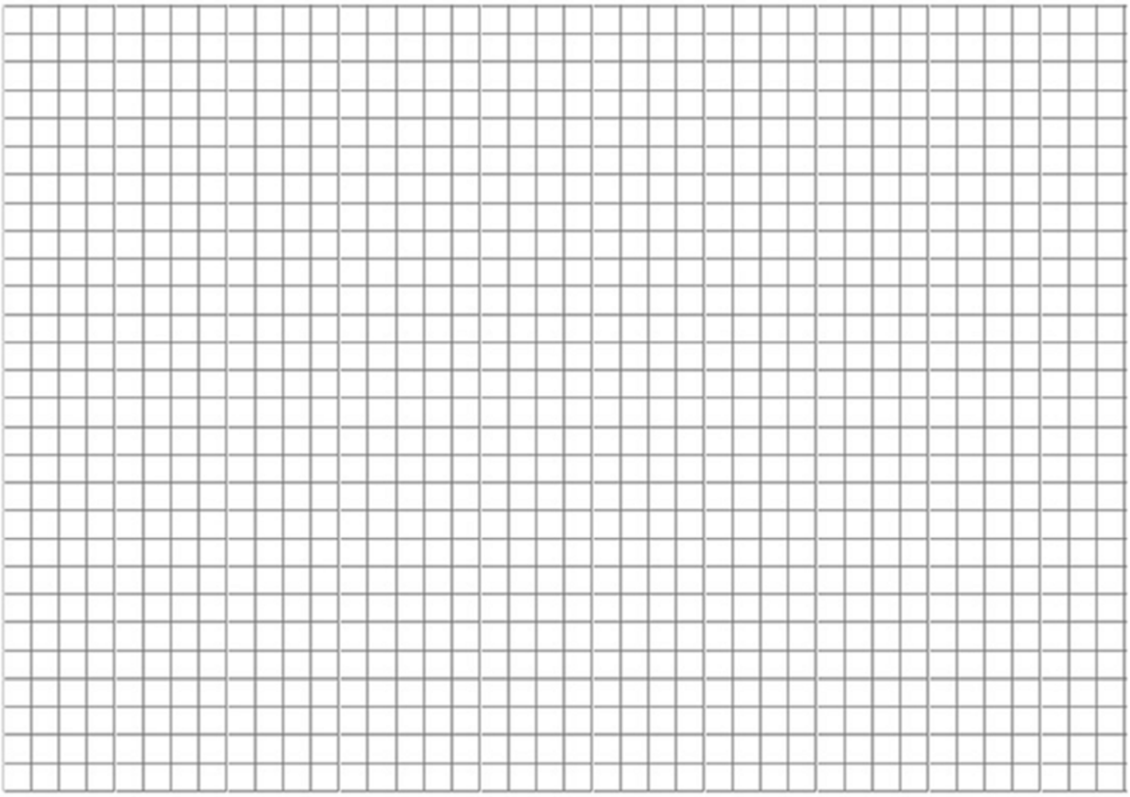
(b) $y = 5 - 2\sin 3x$

(c) $y = 3\sin 2\left(x - \frac{\pi}{2}\right)$



(d) $y = 4\sin(\pi + 3x) - 1$

(e) $y = 2\tan\left(x + \frac{\pi}{2}\right)$



(f) $y = -\tan(2x - \pi) - 3$

QUESTION 5 – EXAM 1

Sketch $f(x) = 3\sin 2\left(x - \frac{\pi}{4}\right) + 2$ for $0 \le x \le 2\pi$ showing all transformations required to obtain the graph of $f(x)$ from $g(x) = \sin x$.

Solution

SKETCHING TRIGONOMETRIC FUNCTIONS

QUESTION 6 – EXAM 1
Sketch the graphs of the following functions showing one complete cycle.

(a) $y = -4\sin x + 2$

Amplitude:

Midline (Vertical Shift):

Maximum:

Minimum:

Period:

Horizontal Shift:

Distance Between Points $\left(\dfrac{Period}{4}\right)$:

(b) $f(x) = \frac{1}{10}\cos 2\left(x + \frac{\pi}{4}\right)$

Amplitude:

Midline (Vertical Shift):

Maximum:

Minimum:

Period:

Horizontal Shift:

Distance Between Points $\left(\frac{Period}{4}\right)$:

(c) $f(x) = -\tan 2\left(x - \frac{\pi}{2}\right)$

Amplitude:

Midline (Vertical Shift):

Maximum:

Minimum:

Period:

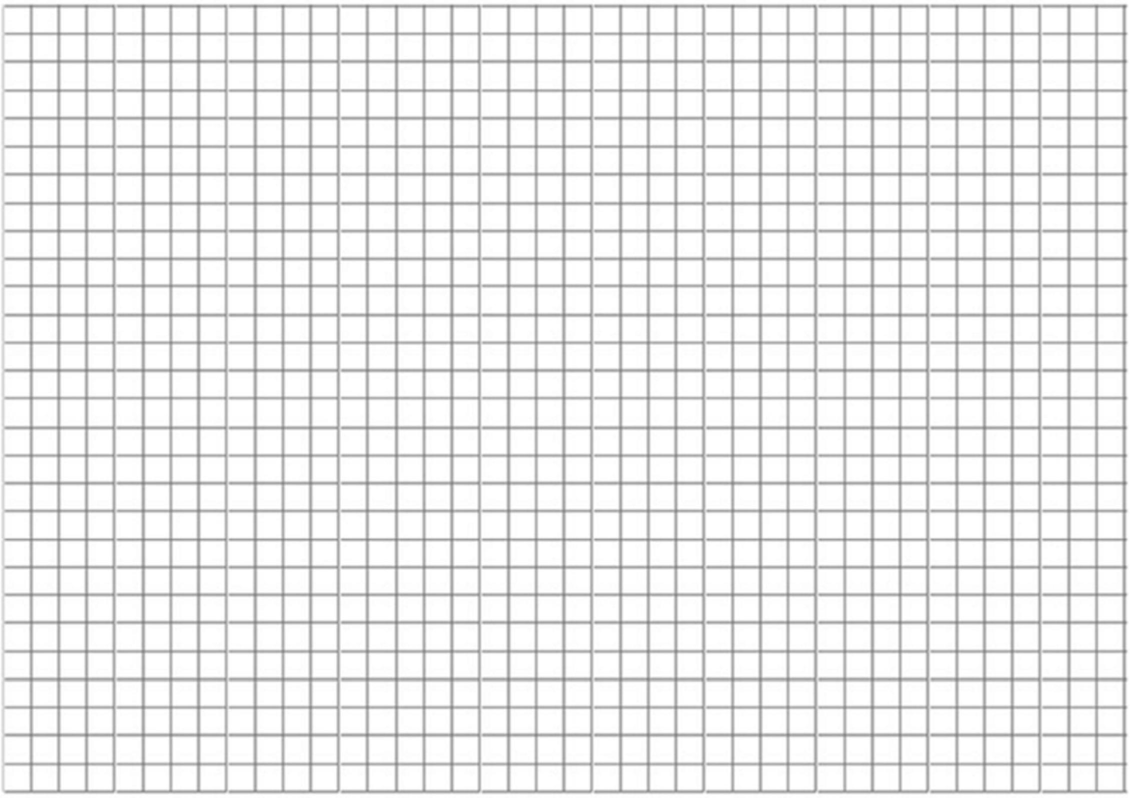
Horizontal Shift:

Distance Between Points $\left(\frac{Period}{4}\right)$:

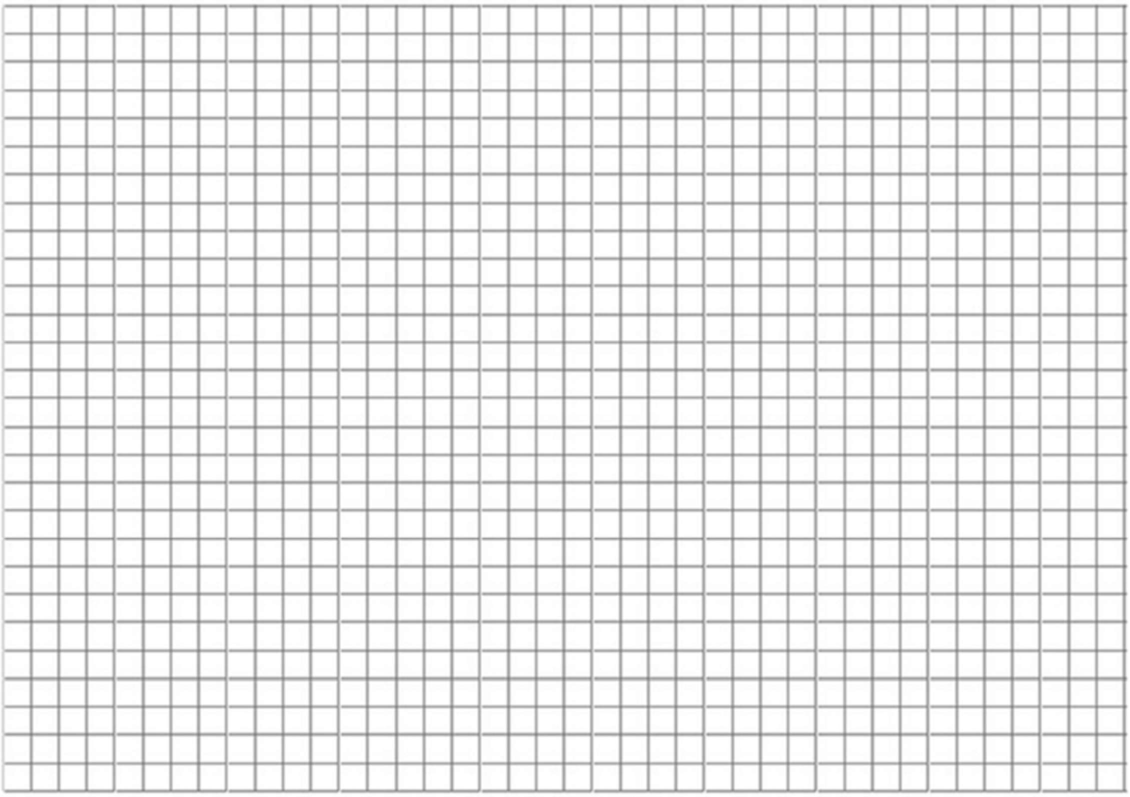
QUESTION 7 – EXAM 1
Sketch one complete cycle of the following functions.

(a) $y = 3\sin\left(2t + \dfrac{\pi}{3}\right)$

(b) $y = 5\cos\left(4x - \frac{\pi}{2}\right) + 1$



(c) $y = 4\sin(-2x - \pi) - 3$



(d) $y=\frac{1}{2}\tan(\pi - x)+\pi$

DETERMINING THE EQUATION OF A TRIGONOMETRIC FUNCTION

$y = a\sin n(\theta - \in) + k \qquad y = a\cos n(\theta - \in) + k \qquad y = a\tan n(\theta - \in) + k$

METHOD:

Step 1: Look carefully at the general given form and identify the possible transformations. eg. $y = a\sin(\theta - \in)$ indicates that there is a horizontal translation, dilation from the θ axis and possibly a reflection (if a is negative).

Step 2: Use features of the graph such as amplitude, period etc to solve for as many unknowns as possible.

The **amplitude** is equal to half the distance between the maximum and minimum values.

i.e. $|a| = \dfrac{y_{\max} - y_{\min}}{2}$

The **midline** is the vertical translation, k.

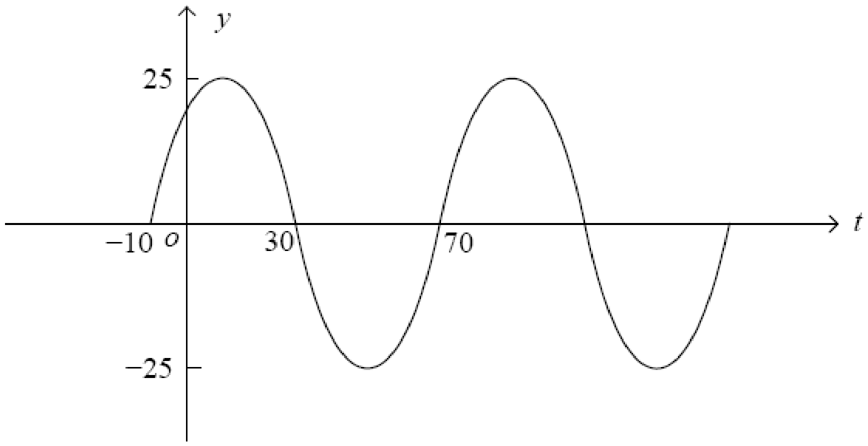
The horizontal translation represents the distance that the first point on the standard curve has been shifted from the Y axis.

The **period**: Read the distance required to complete **ONE** cycle off the horizontal axis. Alternatively, the period is $\dfrac{2\pi}{|n|}$ for the sine and cosine functions and $\dfrac{\pi}{|n|}$ for the tangent function.

To find the equation(s) of the **tangent asymptote(s),** let the angle equal $\pm\dfrac{\pi}{2}$ and solve.

Step 3: Substitute the coordinates of any given (or obvious) points to identify the values of the remaining variables.

QUESTION 8 – EXAM 2
The equation describing the graph below is in the form $y = A\sin(Bt + C)$. Find the values of A, B and C.

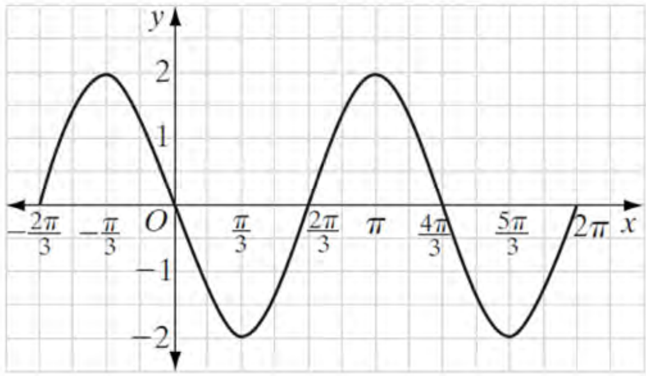


Solution

QUESTION 9 – EXAM 2
Write an equation for the graph below in the form

(a) $y = a\cos b\left(x+c\right)+d$ **.**

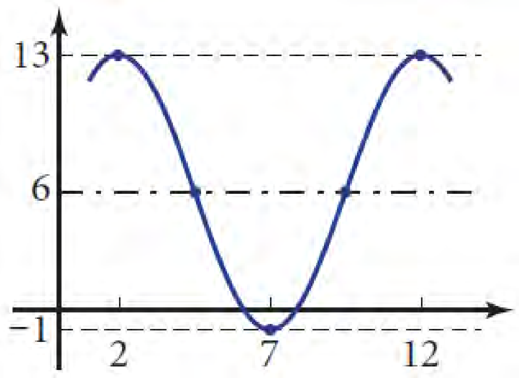
(b) $y = a\sin b\left(x+c\right)+d$



Solution

QUESTION 10 – EXAM 1

The equation describing the graph below is in the form $y = A\cos B(x - D) + C$. Find the equation.



Solution

SOLVING TRIGONOMETRIC EQUATIONS

The values of important Reference Angles are given below:

Angle (θ)	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin\theta$	$\frac{\sqrt{0}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2}$
$\cos\theta$	$\frac{\sqrt{4}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{0}}{2}$
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\frac{\sqrt{3}}{1}$	Undefined

Note: In the first examination paper for Mathematical Methods, the Examiners will assume that the exact values above are known. Learn these values off by heart.

Angles at the Axes

$\sin(0)=0$	$\cos(0)=1$	$\tan(0)=0$
$\sin\left(\frac{\pi}{2}\right)=1$	$\cos\left(\frac{\pi}{2}\right)=0$	$\tan\left(\frac{\pi}{2}\right)=undefined$
$\sin(\pi)=0$	$\cos(\pi)=-1$	$\tan(\pi)=0$
$\sin\left(\frac{3\pi}{2}\right)=-1$	$\cos\left(\frac{3\pi}{2}\right)=0$	$\tan\left(\frac{3\pi}{2}\right)=undefined$
$\sin(2\pi)=0$	$\cos(2\pi)=1$	$\tan(2\pi)=0$

SOLVING TRIGONOMETRIC EQUATIONS

Step 1: Write all expressions in terms of one trigonometric function.

Step 2: Transpose the given equation so that the trigonometric expression (and the angle) is on one side of the equation, and the constants are located on the other side of the equation.

Step 3: Use the sign in front of the constant on the right-hand side to determine the quadrants in which the solutions are to lie. (Use **CAST**)

Step 4: Calculate the first quadrant solution. If the exact value cannot be determined:

Press Inverse Sin, Cos or Tan of the number on the right-hand side of the equation (but ignore the sign).

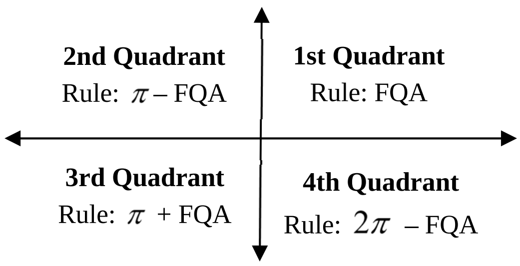
For example: $Sin^{-1}(number\ on\ RHS\ of\ \ equation\ but\ ignore\ the\ sign)$

(Ensure that the calculator is in Radian Mode).

Step 5: Solve for the variable (usually x or θ). Let the angle equal the rule describing angles in the quadrants in which the solutions are to lie.

Note: First Quadrant Angle = FQA

Let angle = FQA if solution lies in 1st Quadrant.
Let angle = $\pi - FQA$ if solution lies in 2nd Quadrant.
Let angle = $\pi + FQA$ if solution lies in 3rd Quadrant.
Let angle = $2\pi - FQA$ if solution lies in 4th Quadrant.



Step 6: Evaluate all possible solutions by observing the given domain. This is accomplished by adding or subtracting the **PERIOD** to each of the solutions, until the angles fall outside the given domain.

For sine and cosine functions: $Period = \dfrac{2\pi}{|The\ number\ in\ \ front\ of\ \ the\ variable|}$

For tangent functions: $Period = \dfrac{\pi}{|The\ number\ in\ \ front\ of\ \ the\ variable|}$

Always look closely at the brackets in the given domain and consider whether the upper and lower limits can be included in your solutions.

DO NOT discard any solution until the final step.

Step 7: Eliminate solutions that do not lie across specified domain.

Note: Students may also solve trigonometric equations by rearranging the domain.

QUESTION 11 – EXAM 1
Solve the following equations for $x \in [-\pi, 2\pi]$.

(a) $3\cos x + 2 = 1$

(b) $\sin x + \sqrt{2} = -\sin x$

(c) $4\tan x - 7 = 3\tan x - 6$

(d) $7\cos x = 5\cos x + \sqrt{3}$

QUESTION 12 – EXAM 2

Solve the following equations across $[0, 2\pi]$.

(a) $\sin 4\theta = \frac{\sqrt{3}}{2}$

(b) $2\cos 3x - 1 = 0$

QUESTION 13 – EXAM 1

Solve the following equations across $[0, 2\pi]$.

(a) $2\sin\left(x-\frac{\pi}{4}\right)+1=2$.

(b) $2\sin\left(\frac{x}{2}+\frac{\pi}{3}\right)+\sqrt{3}=0$.

(c) $-2\cos 2\left(x-\frac{\pi}{6}\right)=\sqrt{2}$.

QUESTION 14 – EXAM 2

(a) Using algebra, find the solution(s) to $2\cos\left(2x+\frac{\pi}{3}\right)+\sqrt{3}=0$, $x\in[-\pi, 2\pi]$.

(b) Hence find $2\cos\left(2x+\frac{\pi}{3}\right)+\sqrt{3}<0$.

Solution

QUESTION 15 – EXAM 1

Given that $f(x)=50+10\sin\frac{\pi}{12}(t-8)$ find $f(x)=60$, $(0,60)$.

Solution

QUESTION 16 – EXAM 2

$f(x)=2.325\sin\frac{\pi}{6}(t-2.667)+12.155$. Solve $f(x)=13$ for $0\le t\le 12$. State your answer correct to 2 decimal places.

Solution

QUESTION 17 – EXAM 2

Solve $35-28\cos\left(\frac{\pi t}{6.2}\right)=3.5$ for $1<t<9$. State your answer correct to 2 decimal places.

Solution

SOLUTIONS

QUESTION 1 – EXAM 1

(a) $f(x) = 3\cos\left(2x + \frac{\pi}{4}\right)$, $g(x) = \cos x$.

Dilation from the Y axis **Yes** **No** **Factor:** $\frac{1}{2}$	**Dilation from the X axis** **Yes** **No** **Factor:** 3
Reflection in the Y axis: **Yes** **No**	**Reflection in the X axis:** **Yes** **No**
Translation parallel to the X axis **Yes** **No** **Number of Units:** **+ve direction** **-ve direction**	**Translation parallel to the Y axis** **Yes** **No** **Number of Units:** **+ve direction** **-ve direction**

(b) $f(x) = 1 - 3\sin(2x - 9)$, $g(x) = \sin x$.

Dilation from the Y axis **Yes** **No** **Factor:** $\frac{1}{2}$	**Dilation from the X axis** **Yes** **No** **Factor:** 3
Reflection in the Y axis: **Yes** **No**	**Reflection in the X axis:** **Yes** **No**
Translation parallel to the X axis **Yes** **No** **Number of Units:** 4.5 **+ve direction** **-ve direction**	**Translation parallel to the Y axis** **Yes** **No** **Number of Units:** 1 **+ve direction** **-ve direction**

(c) $f(x) = 2\tan\left(\frac{\pi}{4} - \frac{x}{3}\right) - 5$, $g(x) = \tan x$.

Dilation from the Y axis **Yes** **No** **Factor: 3**	**Dilation from the X axis** **Yes** **No** **Factor: 2**
Reflection in the Y axis: **Yes** **No**	**Reflection in the X axis:** **Yes** **No**
Translation parallel to the X axis **Yes** **No** **Number of Units:** $\frac{3\pi}{4}$ **+ve direction** **-ve direction**	**Translation parallel to the Y axis** **Yes** **No** **Number of Units: 5** **+ve direction** **-ve direction**

QUESTION 2 – EXAM 2

Answer is D

f(x) dilated by 2 from x axis : $2\cos(x)$

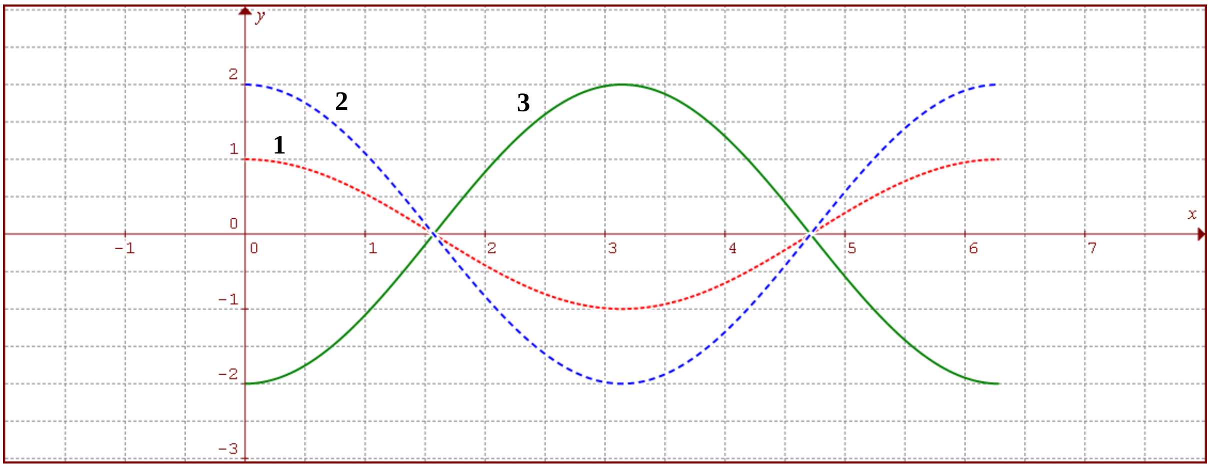
f(x) dilated by 3 from Y axis : $2\cos\left(\frac{x}{3}\right)$

Horizontal translation : $2\cos\frac{1}{3}(x-4)$

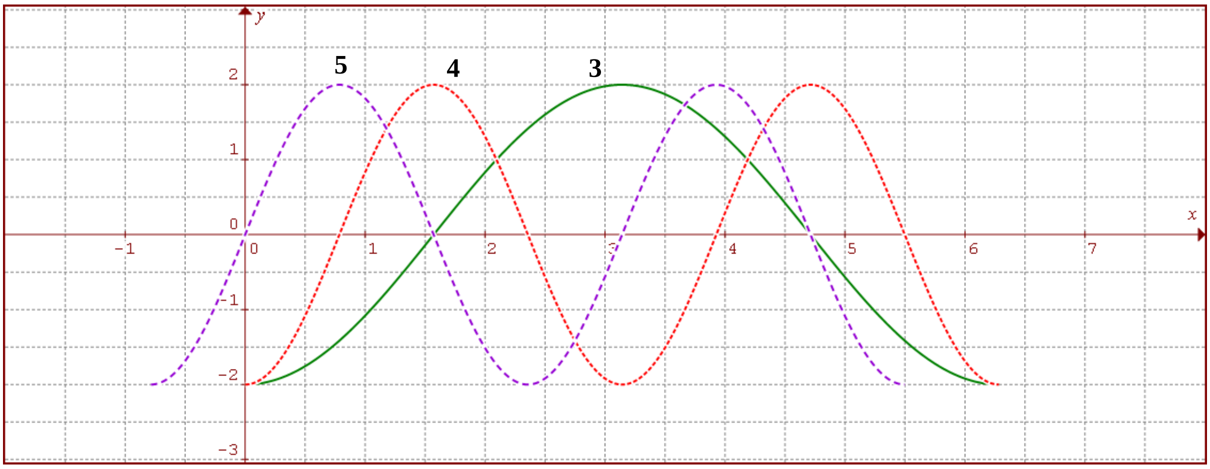
Vertical translation : $2\cos\frac{1}{3}(x-4) - 2$

QUESTION 4 – EXAM 1

(a) $y = -2\cos\left(2x + \frac{\pi}{2}\right)$

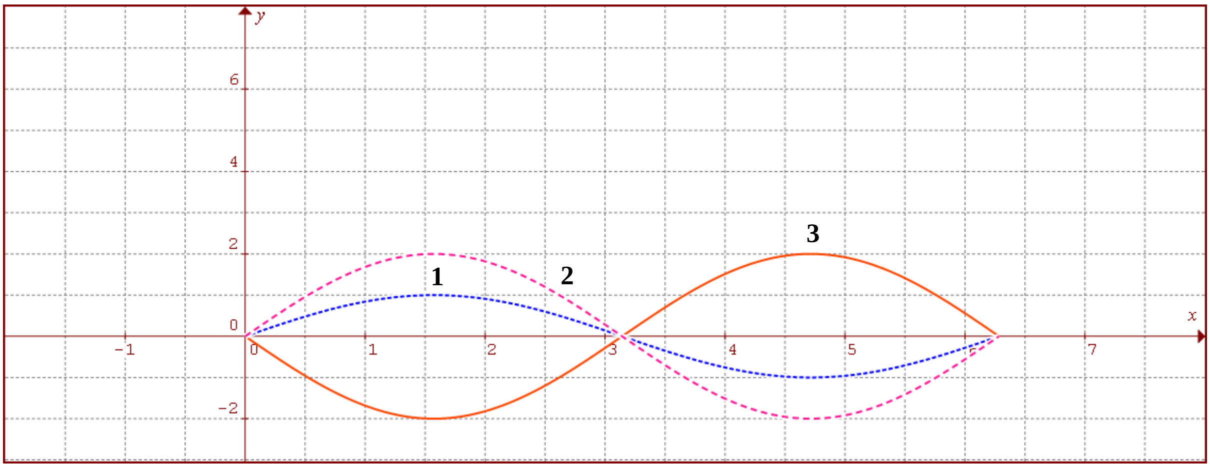


1. $y = \cos x$
2. $y = 2\cos x$
3. $y = -2\cos x$

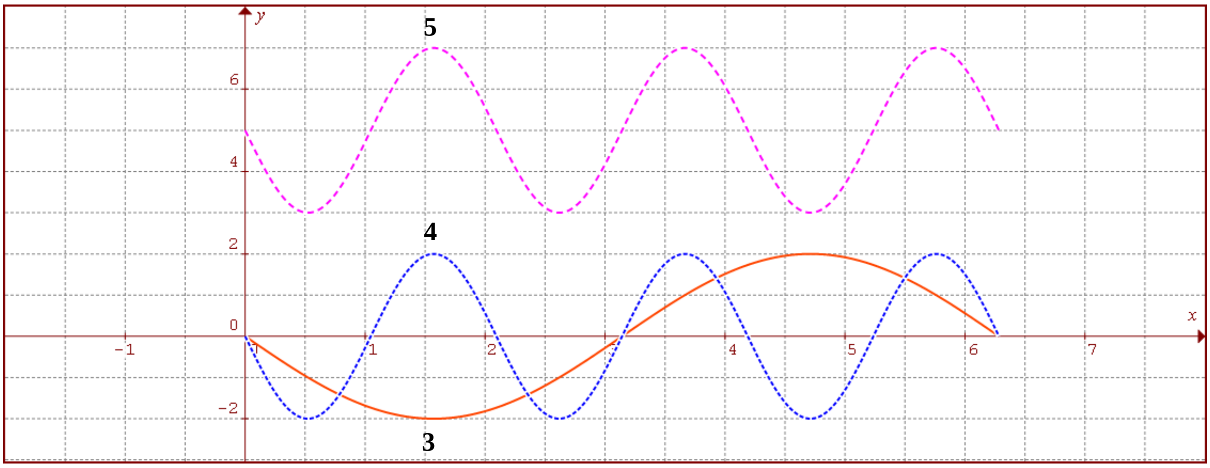


3. $y = -2\cos x$
4. $y = -2\cos 2x$
5. $y = -2\cos\left(2x + \frac{\pi}{2}\right)$

(b) $y = 5 - 2\sin 3x$

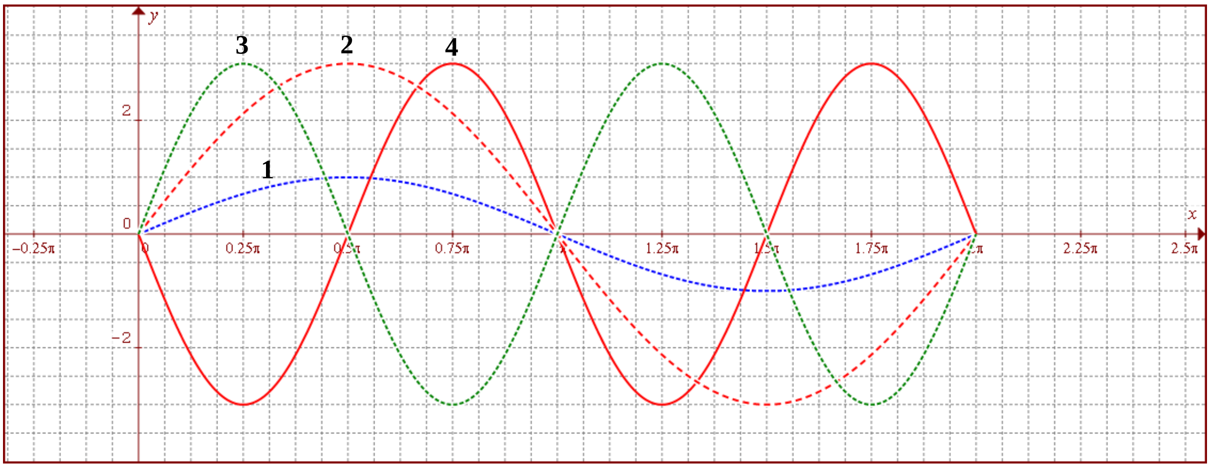


1. $y = \sin x$
2. $y = 2\sin x$
3. $y = -2\sin x$



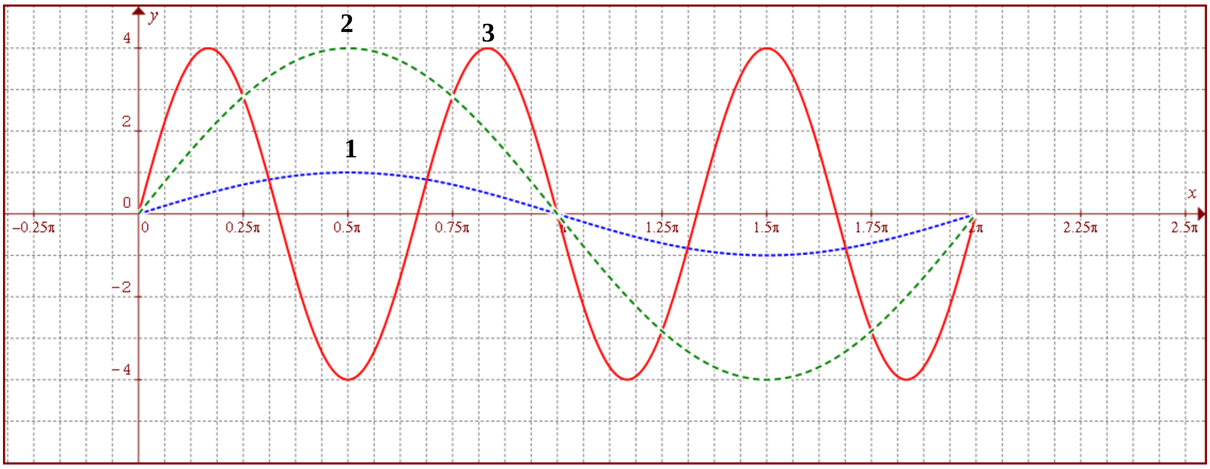
3. $y = -2\sin x$
4. $y = -2\sin 3x$
5. $y = 5 - 2\sin 3x$

(c) $y = 3\sin 2\left(x - \frac{\pi}{2}\right)$

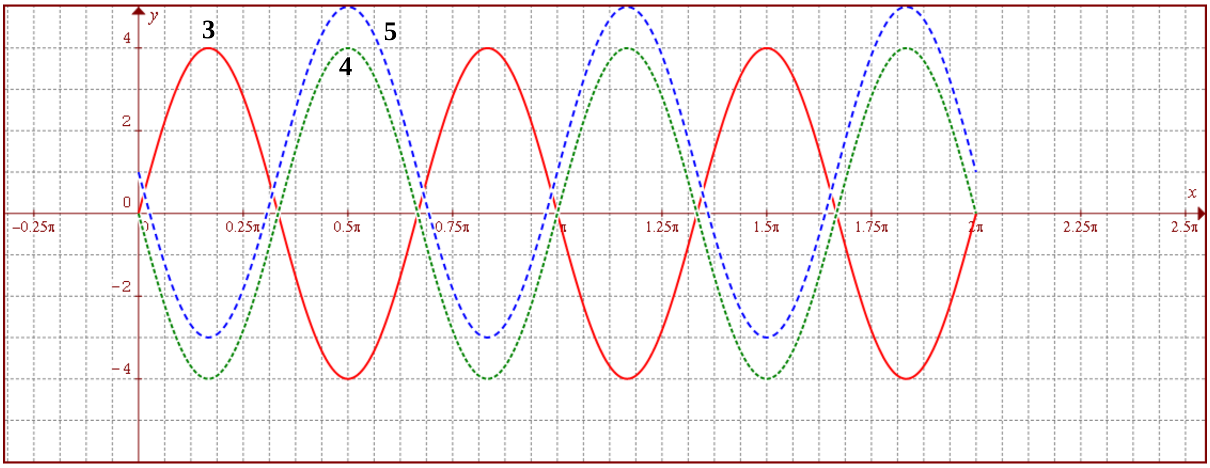


1. $y = \sin x$
2. $y = 3\sin x$
3. $y = 3\sin 2x$
4. $y = 3\sin 2\left(x - \frac{\pi}{2}\right)$

(d) $y = 4\sin(\pi + 3x) - 1$

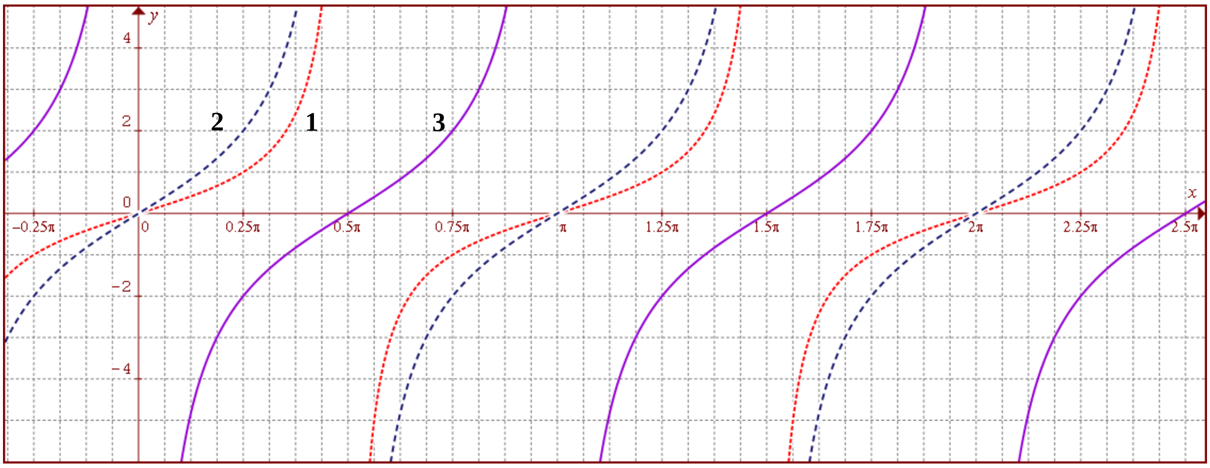


1. $y = \sin x$
2. $y = 4\sin x$
3. $y = 4\sin 3x$



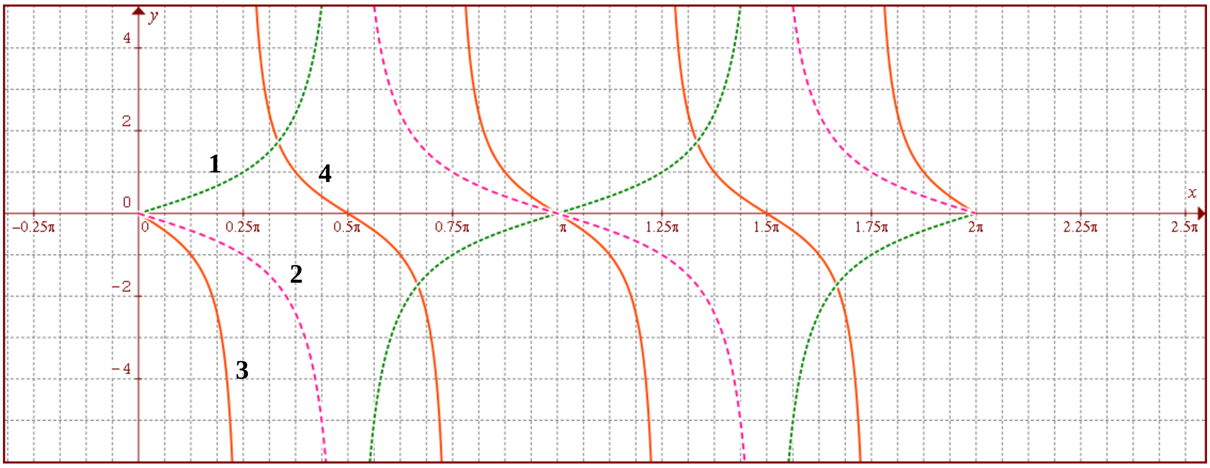
3. $y = 4\sin 3x$
4. $y = 4\sin(\pi + 3x)$
5. $y = 4\sin(\pi + 3x) - 1$

(e) $y = 2\tan\left(x + \dfrac{\pi}{2}\right)$

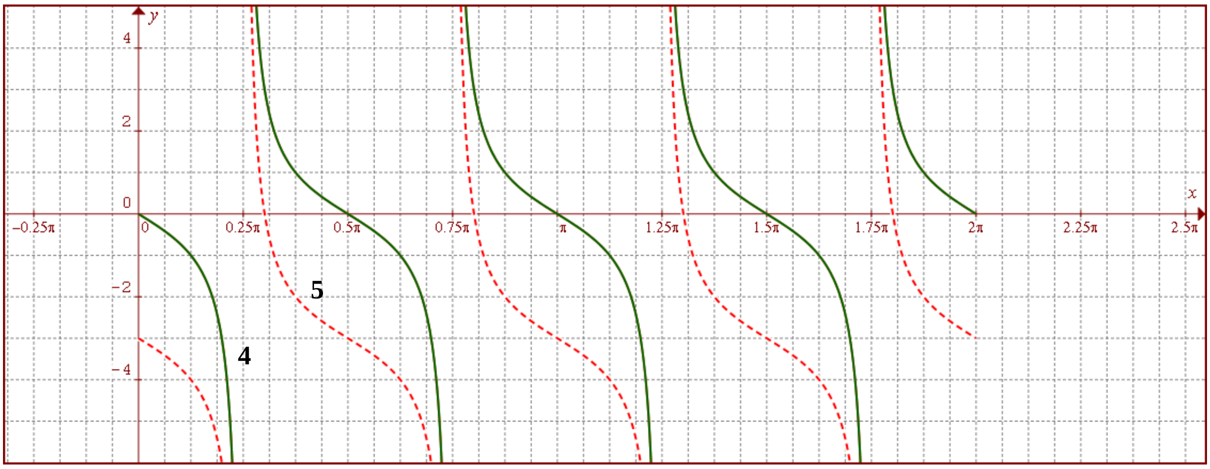


1. $y = \tan x$
2. $y = 2\tan x$
3. $y = 2\tan\left(x + \dfrac{\pi}{2}\right)$

(f) $y = -\tan(2x - \pi) - 3$



1. $y = \tan x$
2. $y = -\tan x$
3. $y = -\tan 2x$
4. $y = -\tan(2x - \pi)$

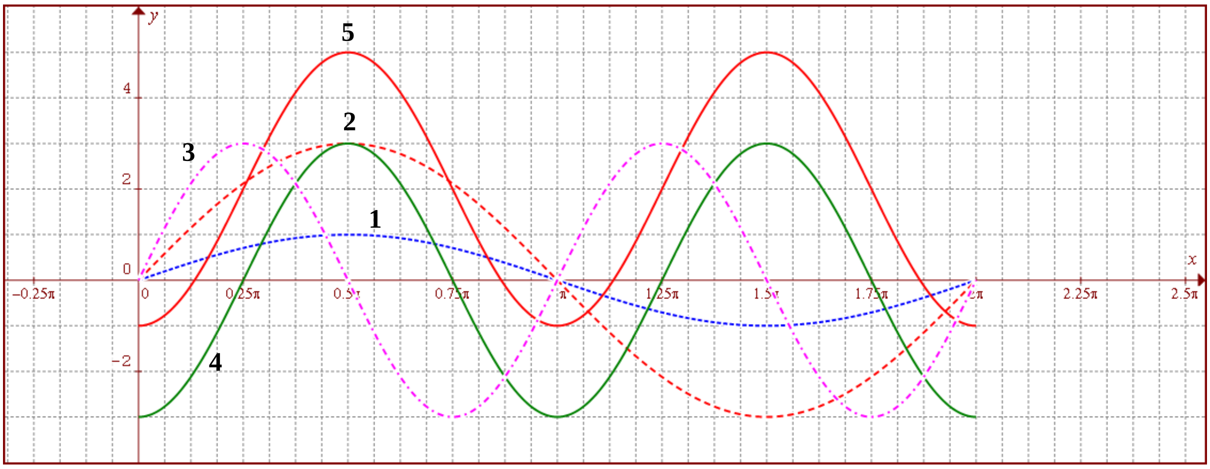


4. $y = -\tan(2x - \pi)$
5. $y = -\tan(2x - \pi) - 3$

QUESTION 5 – EXAM 1

Sketch $f(x)=3\sin 2\left(x-\frac{\pi}{4}\right)+2$ for $0\le x\le 2\pi$ showing all transformations required to obtain the graph of $f(x)$ from $g(x)=\sin x$.

Solution

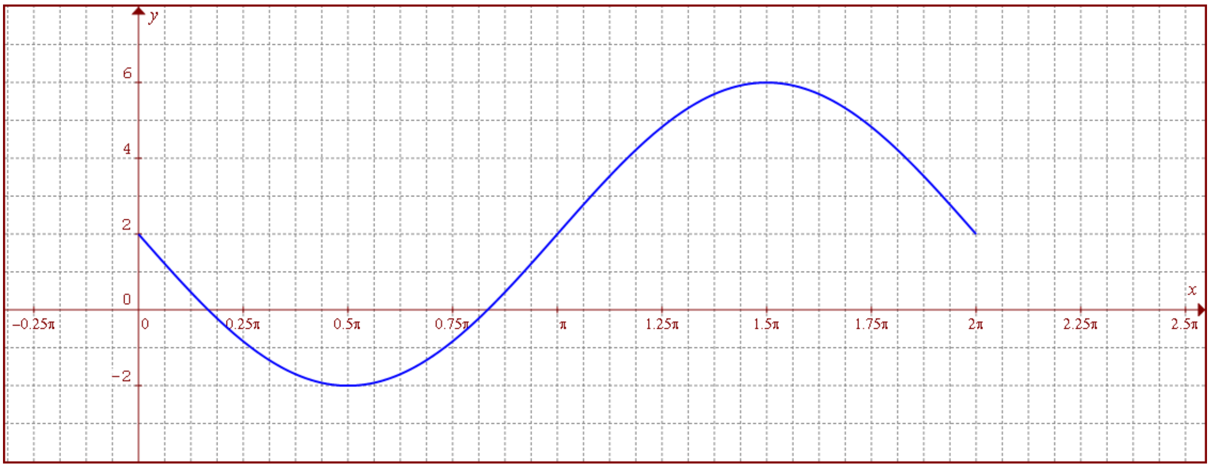


1. $f(x)=\sin x$
2. $f(x)=3\sin x$
3. $f(x)=3\sin 2x$
4. $f(x)=3\sin 2\left(x-\frac{\pi}{4}\right)$
5. $f(x)=3\sin 2\left(x-\frac{\pi}{4}\right)+2$

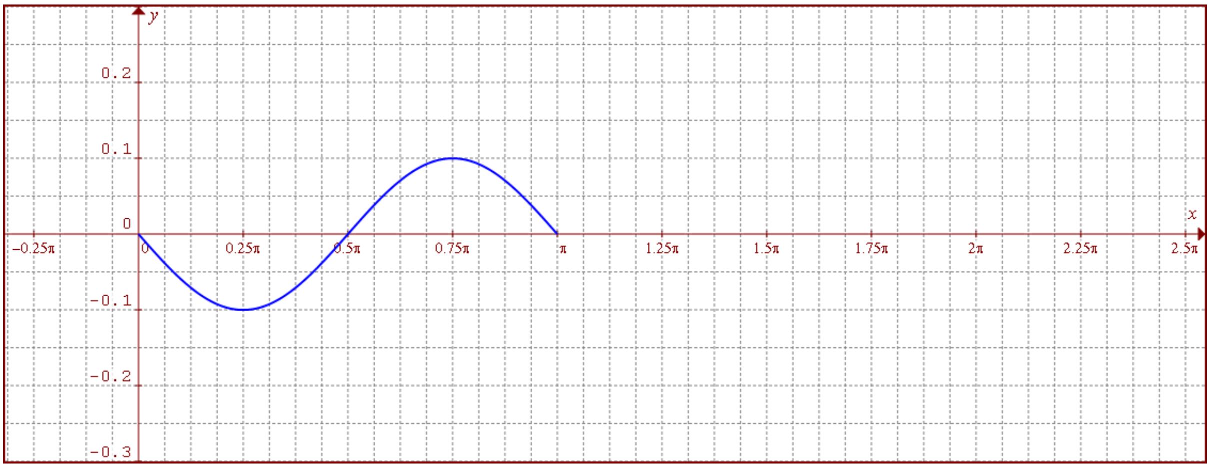
QUESTION 6 – EXAM 1

Sketch the graphs of the following functions showing one complete cycle.

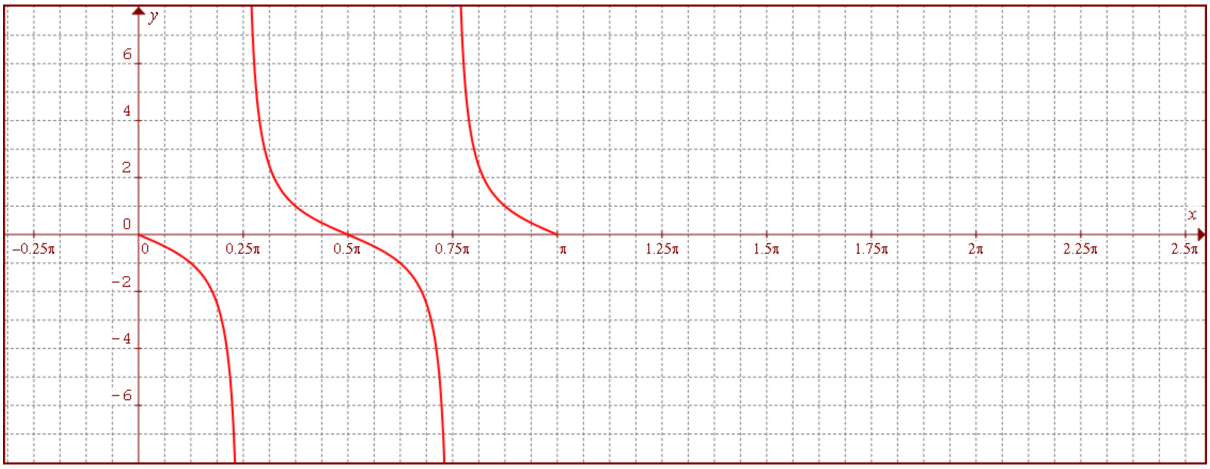
(a) $y = -4\sin x + 2$



(b) $f(x) = \frac{1}{10}\cos 2\left(x + \frac{\pi}{4}\right)$

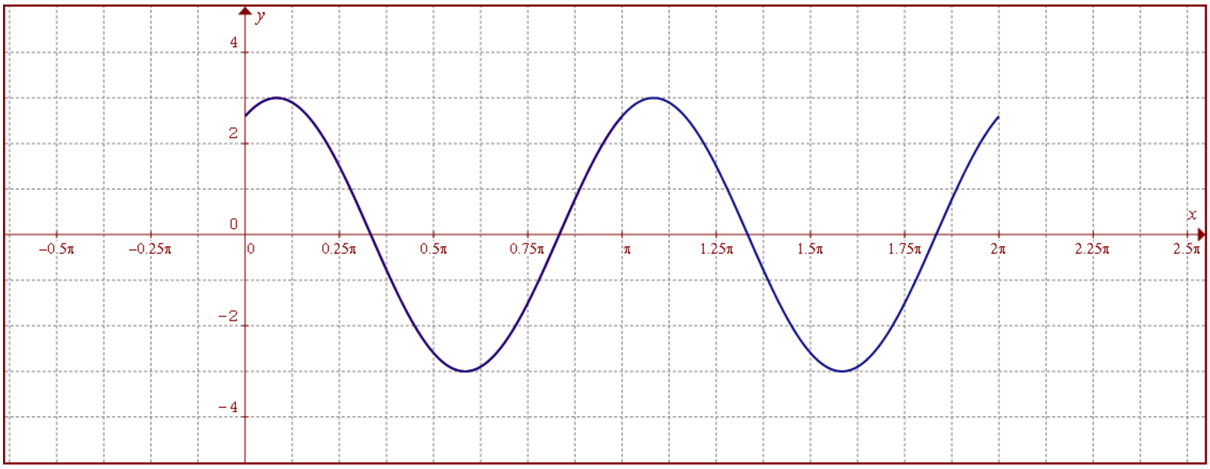


(c) $f(x) = -\tan 2\left(x - \frac{\pi}{2}\right)$

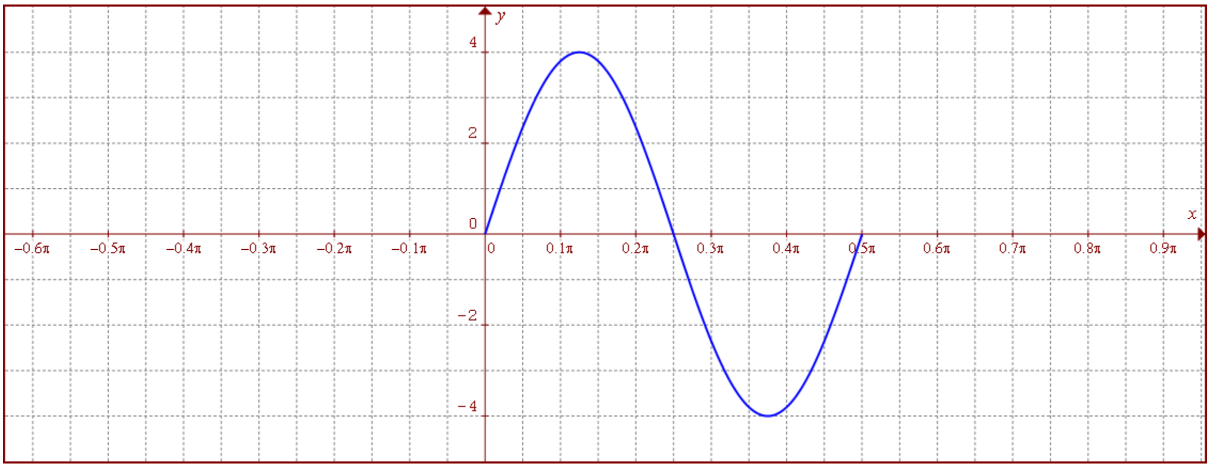


QUESTION 7 – EXAM 1

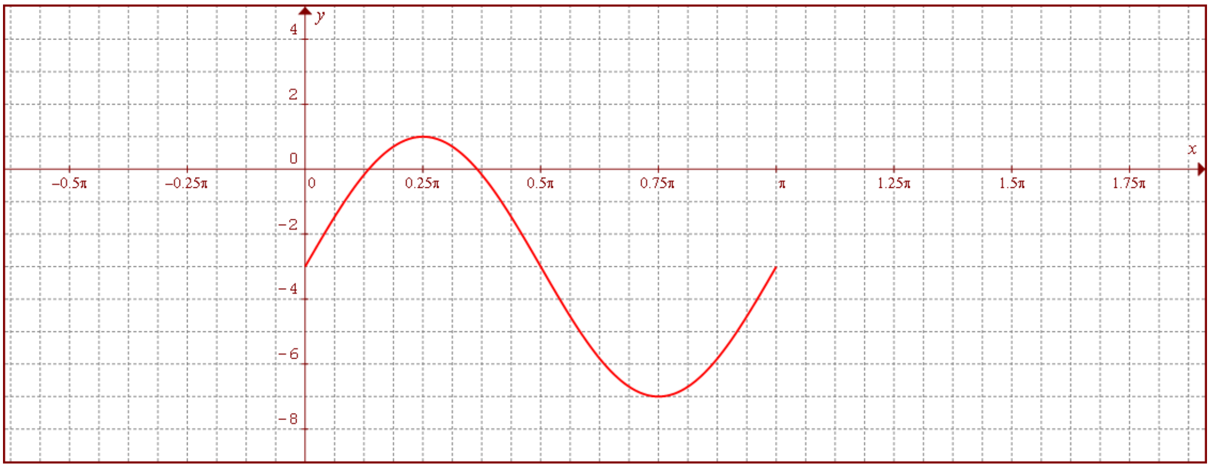
(a) $y = 3\sin\left(2t + \frac{\pi}{3}\right)$



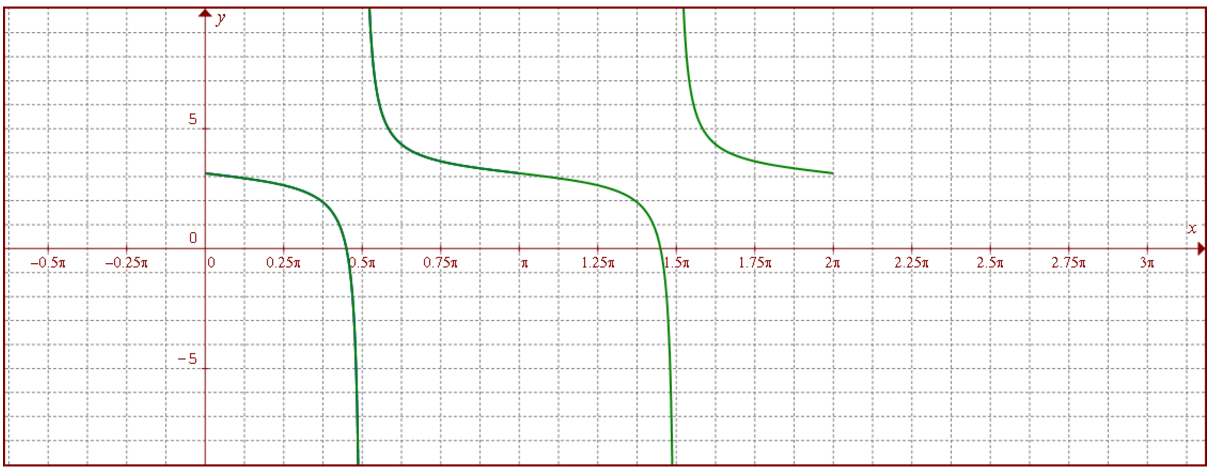
(b) $y = 5\cos\left(4x - \frac{\pi}{2}\right) + 1$



(c) $y = 4\sin(-2x - \pi) - 3$



(d) $y = \frac{1}{2}\tan(\pi - x) + \pi$



QUESTION 8 – EXAM 2

Amplitude = 25

There is no reflection in the x axis $\therefore A = 25$

$$\text{Period} = \frac{2\pi}{B} = 80$$

$$\therefore B = \frac{2\pi}{80} = \frac{\pi}{40}$$

Translation = 10 units to left

$$\therefore -\frac{C}{B} = {}^{-}10$$

$$C = 10B$$

$$C = \frac{10\pi}{40} = \frac{\pi}{4}$$

$$\therefore y = 25\sin\left(\frac{\pi}{40}t + \frac{\pi}{4}\right)$$

QUESTION 9 – EXAM 2

(a) $y = a\cos b\left(x+c\right)+d$

$$\text{Amplitude} = \frac{2-(-2)}{2} = 2 \qquad \therefore a = 2$$

$$\text{Period} = \pi - -\frac{\pi}{3} = \frac{4\pi}{3}$$

$$\frac{2\pi}{b} = \frac{4\pi}{3}$$

$$\therefore b = \frac{6\pi}{4\pi} = \frac{3}{2}$$

Translation = $\frac{\pi}{3}$ units to left $\therefore c = \frac{\pi}{3}$

$$\therefore y = 2\cos\frac{3}{2}\left(x + \frac{\pi}{3}\right)$$

No vertical translation $\therefore d = 0$

(b) $y = a\sin b(x+c)+d$

Amplitude $= \frac{2 - -2}{2} = 2 \quad \therefore a = 2$

$b = \frac{3}{2}$

$d = 0$

Translation $= \frac{2\pi}{3}$ to the right $\therefore c = -\frac{2\pi}{3}$

$\therefore y = 2\sin\frac{3}{2}\left(x - \frac{2\pi}{3}\right)$

QUESTION 10 – EXAM 1

Amplitude $= \frac{13 - -1}{2} = \frac{14}{2} = 7 \quad \therefore A = 7$

Period $= 10 = \frac{2\pi}{B}$

$\therefore B = \frac{2\pi}{10} = \frac{\pi}{5}$

Vertical translation $= 6$ (as midline is at $y = 6$)

$\therefore C = 6$

Horizontal translation $=$ 2 units to right

$\therefore D = 2$

$\therefore y = 7\cos\frac{\pi}{5}(x - 2) + 6$

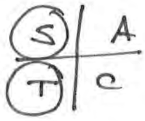
QUESTION 11 – EXAM 1

(a) $3\cos x + 2 = 1$

$3\cos x + 2 = 1$

$3\cos x = -1$

$\cos x = -\frac{1}{3}$



Reference Angle = 1.23096

$x = \pi - 1.23096,\ \pi + 1.23096$

$x = 1.96079,\ 4.37255$

$x = -1.91063,\ 1.96079,\ 4.37255$

$x = -1.91,\ 1.96,\ 4.37$

(b) $\sin x + \sqrt{2} = -\sin x$

$\sin x + \sqrt{2} = -\sin x$

$2\sin x = -\sqrt{2}$

$\sin x = -\frac{\sqrt{2}}{2}$

Reference Angle = $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$

$x = \pi + \frac{\pi}{4},\ 2\pi - \frac{\pi}{4}$

$x = \frac{5\pi}{4},\ \frac{7\pi}{4}$

Add/subtract period = 2π

$x = -\frac{3\pi}{4},\ \frac{-\pi}{4},\ \frac{5\pi}{4},\ \frac{7\pi}{4}$

(c) $4\tan x - 7 = 3\tan x - 6$

$4\tan x - 7 = 3\tan x - 6$

$\tan x = 1$

Reference Angle $= \frac{\pi}{4}$

$x = \frac{\pi}{4}, \quad \pi + \frac{\pi}{4}$

$x = \frac{\pi}{4}, \quad \frac{5\pi}{4}$

Add/subtract period $= \pi$

$x = -\frac{3\pi}{4}, \frac{\pi}{4}, \frac{5\pi}{4}$

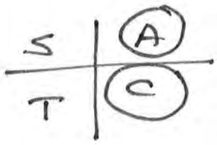
(d) $7\cos x = 5\cos x + \sqrt{3}$

$7\cos x = 5\cos x + \sqrt{3}$

$2\cos x = \sqrt{3}$

$\cos x = \frac{\sqrt{3}}{2}$

Reference Angle $= \frac{\pi}{6}$



$x = \frac{\pi}{6}, \quad 2\pi - \frac{\pi}{6}$

$x = \frac{\pi}{6}, \quad \frac{11\pi}{6}$

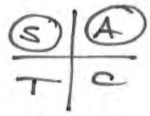
Add/subtract period $= \frac{12\pi}{6}$

$x = -\frac{\pi}{6}, \frac{\pi}{6}, \frac{11\pi}{6}$

QUESTION 12 – EXAM 2

(a) $\sin 4\theta = \frac{\sqrt{3}}{2}$

$\sin 4\theta = \frac{\sqrt{3}}{2}$



Reference Angle $= \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$

$4\theta = \frac{\pi}{3}, \ \pi - \frac{\pi}{3}$

$4\theta = \frac{\pi}{3}, \ \frac{2\pi}{3}$

$\theta = \frac{\pi}{12}, \ \frac{2\pi}{12}$

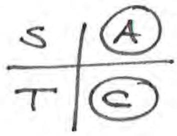
$\theta = \frac{\pi}{12}, \ \frac{\pi}{6}$

(b) $2\cos 3x - 1 = 0$

$2\cos 3x - 1 = 0$

$2\cos 3x = 1$

$\cos 3x = \frac{1}{2}$



Reference Angle $= \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

$3x = \frac{\pi}{3}, \ 2\pi - \frac{\pi}{3}$

$3x = \frac{\pi}{3}, \ \frac{5\pi}{3}$

$x = \frac{\pi}{9}, \ \frac{5\pi}{9}$

Add/subtract period $= \frac{2\pi}{3} = \frac{6\pi}{9}$

$x = \frac{\pi}{9}, \ \frac{5\pi}{9}, \ \frac{7\pi}{9}, \ \frac{11\pi}{9}, \ \frac{13\pi}{9}, \ \frac{17\pi}{9}$

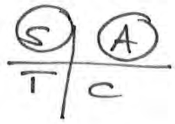
QUESTION 13 – EXAM 1

(a) $2\sin\left(x - \frac{\pi}{4}\right) + 1 = 2$.

$$2\sin\left(x - \frac{\pi}{4}\right) + 1 = 2$$

$$2\sin\left(x - \frac{\pi}{4}\right) = 1$$

$$\sin\left(x - \frac{\pi}{4}\right) = \frac{1}{2}$$



Reference Angle $= \frac{\pi}{6}$

$$x - \frac{\pi}{4} = \frac{\pi}{6},\ \pi - \frac{\pi}{6}$$

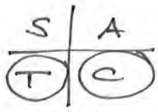
$$x - \frac{3\pi}{12} = \frac{2\pi}{12},\ \frac{10\pi}{12}$$

$$x = \frac{5\pi}{12},\ \frac{13\pi}{12}$$

(b) $2\sin\left(\frac{x}{2} + \frac{\pi}{3}\right) + \sqrt{3} = 0$.

$$2\sin\left(\frac{x}{2} + \frac{\pi}{3}\right) + \sqrt{3} = 0$$

$$\sin\left(\frac{x}{2} + \frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$



Reference Angle $= \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$

$$\frac{x}{2} + \frac{\pi}{3} = \pi + \frac{\pi}{3},\ 2\pi - \frac{\pi}{3}$$

$$\frac{x}{2} + \frac{\pi}{3} = \frac{4\pi}{3},\ \frac{5\pi}{3}$$

$$\frac{x}{2} = \frac{3\pi}{3},\ \frac{4\pi}{3}$$

$$x = \frac{6\pi}{3},\ \frac{8\pi}{3}$$

$$x = 2\pi$$

(c) $-2\cos 2\left(x-\frac{\pi}{6}\right)=\sqrt{2}$.

$$-2\cos 2\left(x-\frac{\pi}{6}\right)=\sqrt{2}$$

$$\cos 2\left(x-\frac{\pi}{6}\right)=-\frac{\sqrt{2}}{2}$$

Reference Angle $=\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)=\frac{\pi}{4}$

S | A
T | C

$$2\left(x-\frac{\pi}{6}\right)=\pi-\frac{\pi}{4}\ ,\ \pi+\frac{\pi}{4}$$

$$=\frac{3\pi}{4}\ ,\ \frac{5\pi}{4}$$

$$x-\frac{\pi}{6}=\frac{3\pi}{8}\ ,\ \frac{5\pi}{8}$$

$$x-\frac{4\pi}{24}=\frac{9\pi}{24}\ ,\ \frac{15\pi}{24}$$

$$x=\frac{13\pi}{24}\ ,\ \frac{19\pi}{24}$$

Add | subtract period $=\frac{2\pi}{2}=\frac{24\pi}{24}$

$$x=\frac{13\pi}{24}\ ,\ \frac{19\pi}{24}\ ,\ \frac{37\pi}{24}\ ,\ \frac{43\pi}{24}$$

QUESTION 14 – EXAM 2

(a)

$2\cos\left(2x+\frac{\pi}{3}\right) = -\sqrt{3}$

$\cos\left(2x+\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$

1st Quad Angle = $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$

Quadrants 2 and 3

S | A
T | C

$2x+\frac{\pi}{3} = \pi - \text{1st Quad}$

$= \pi - \frac{\pi}{6}$

$= \frac{5\pi}{6}$

$\therefore 2x = \frac{5\pi}{6} - \frac{2\pi}{6} = \frac{3\pi}{6}$

$\therefore x = \frac{3\pi}{12}$

$2x+\frac{\pi}{3} = \pi + \text{1st Quad}$

$= \pi + \pi/6 = 7\pi/6$

$\therefore 2x = \frac{7\pi}{6} - \frac{2\pi}{6} = 5\pi/6$

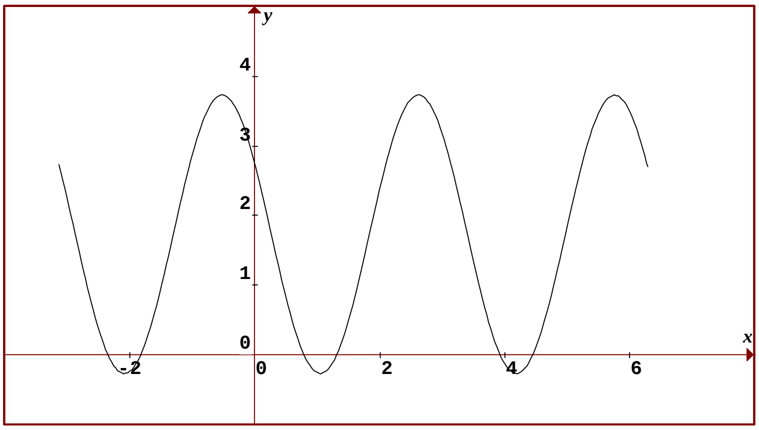
$\therefore x = \frac{5\pi}{12}$

$T = \frac{2\pi}{2} = \pi = \frac{12\pi}{12}$

$x = -\frac{9\pi}{12}, -\frac{7\pi}{12}, \frac{3\pi}{12}, \frac{5\pi}{12}, \frac{15\pi}{12}, \frac{17\pi}{12}$

$x = -\frac{3\pi}{4}, -\frac{7\pi}{12}, \frac{\pi}{4}, \frac{5\pi}{12}, \frac{5\pi}{4}, \frac{17\pi}{12}$

(b)



$$\left\{x: -\frac{3\pi}{4} < x < -\frac{7\pi}{12}\right\} \cup \left\{x: \frac{\pi}{4} < x < \frac{5\pi}{12}\right\} \cup \left\{x: \frac{5\pi}{4} < x < \frac{17\pi}{12}\right\}$$

QUESTION 15 – EXAM 1

$$50 + 10\sin\frac{\pi}{12}(t-8) = 60$$

$$\sin\frac{\pi}{12}(t-8) = -1$$

$$\sin\frac{\pi}{12}(t-8) = \sin\frac{3\pi}{2}$$

$$\therefore \frac{\pi}{12}(t-8) = \frac{3\pi}{2}$$

$$t-8 = \frac{36\pi}{2\pi}$$

$$t-8 = 18$$

$$t = 26$$

Add/subtract period $= \frac{2\pi}{\pi/12} = 24$

$t = 2, 26, 50$

QUESTION 16 – EXAM 2

$$2.325\sin\frac{\pi}{6}(t-2.667) + 12.155 = 13$$

$$2.325\sin\frac{\pi}{6}(t-2.667) = 0.845$$

$$\sin\frac{\pi}{6}(t-2.667) = 0.363 \qquad \begin{array}{c|c} S & A \\ \hline T & C \end{array}$$

Reference Angle $= \sin^{-1}(0.363) = 0.371$

$$\frac{\pi}{6}(t-2.667) = 0.371,\ \pi - 0.371$$

$$t-2.667 = 0.709,\ 5.292$$

$$t = 3.38,\ 7.96$$

QUESTION 17 – EXAM 2

$$35 - 28\cos\left(\frac{\pi}{6.2}\right)t = 3.5$$

$$-28\cos\left(\frac{\pi}{6.2}\right)t = \frac{1}{10}$$

$$\cos\left(\frac{\pi}{6.2}\right)t = -\frac{1}{280}$$

$$\text{Reference Angle} = \cos^{-1}\left(\frac{1}{280}\right) = 1.56722$$

$$\frac{\pi}{6.2}t = \pi - 1.56722,\ \pi + 1.56722$$

$$= 1.57437,\ 4.70882$$

$$= 3.10706,\ 9.29296$$

$$= 3.11$$