Worksheet 4.3 Integrating Special Functions

Section 1 Exponential and Logarithmic Functions

Recall from worksheet 3.10 that the derivative of e^x is e^x. It then follows that the anti derivative of e^x is e^x:

$$\int e^x \, dx = e^x + c$$

In worksheet 3.10 we also discussed the derivative of $e^{f(x)}$ which is $f'(x)e^{f(x)}$. It then follows that

$$\int f'(x)e^{f(x)} \, dx = e^{f(x)} + c$$

where $f(x)$ can be any function. There are other ways of doing such integrations, one of which is by substitution.

Example 1 : Evaluate the indefinite integral $\int 3e^{3x+2} \, dx$.

We recognize that $3 = \frac{d(3x+2)}{dx}$ so that the expression we are integrating has the form $f'(x)e^{f(x)}$. Then

$$\int 3e^{3x+2} \, dx = e^{3x+2} + c$$

Alternatively, we could do it by substitution: let $u = 3x + 2$. Then $du = 3dx$, and

$$\int 3e^{3x+2} \, dx = \int e^u \, du = e^u = e^{3x+2}$$

Note that the integral of the function e^{ax+b} (where a and b are constants) is given by

$$\int e^{ax+b} \, dx = \frac{1}{a}e^{ax+b} + c$$

Example 2 : Find the area under the curve $y = e^{5x}$ between 0 and 2.

$$\begin{aligned} A &= \int_0^2 e^{5x} \, dx \\ &= \left. \frac{1}{5}e^{5x} \right]_0^2 \\ &= \frac{1}{5}e^{10} - \frac{1}{5}e^0 \\ &= \frac{1}{5}(e^{10} - 1) \end{aligned}$$