7.5 Graphing Square Root and Cube Root Functions

Goals • Graph square root and cube root functions.
• Use radical functions to find real-life quantities.

Your Notes

VOCABULARY

Radical function A function containing a radical such as $y = \sqrt{x}$

GRAPHS OF RADICAL FUNCTIONS

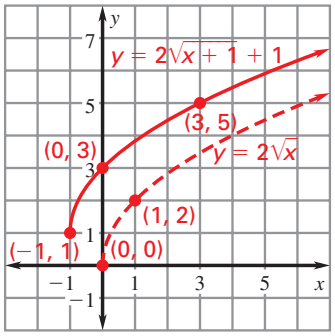
Follow these steps to graph $y = a\sqrt{x - h} + k$ or $y = a\sqrt[3]{x - h} + k$.

Step 1 Sketch the graph of $y = a\sqrt{x}$ or $y = a\sqrt[3]{x}$.

Step 2 Shift the graph h units horizontally and k units vertically.

Example 1 ***Graphing a Square Root Function***

Graph $y = 2\sqrt{x + 1} + 1$.



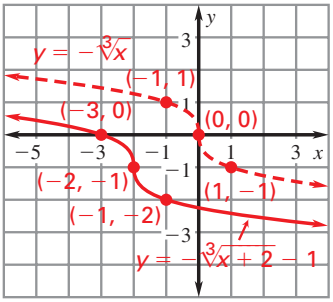
Solution

1. Sketch the graph of $y = 2\sqrt{x}$. Notice that it begins at the origin and passes through (1 , 2).
2. Note that for $y = 2\sqrt{x + 1} + 1$, $h =$ −1 and $k =$ 1 . So, shift the graph left 1 unit and up 1 unit . The result is a graph that passes through the points (−1, 1), (0, 3), and (3, 5) .

Your Notes

Example 2 Graphing a Cube Root Function

Graph $y = -\sqrt[3]{x + 2} - 1$.



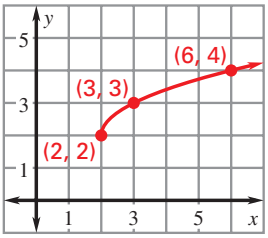
Solution

1. Sketch the graph of $y = -\sqrt[3]{x}$. Notice that it begins at the origin and passes through the points (−1, 1) and (1, −1).

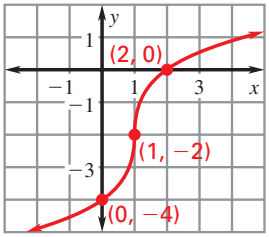
2. Note that for $y = -\sqrt[3]{x + 2} - 1$, $h =$ −2 and $k =$ −1. So, shift the graph left 2 units and down 1 unit. The result is a graph that passes through the points (−3, 0), (−2, −1), and (−1, −2).

✔ ***Checkpoint*** **Graph the function.**

1. $y = \sqrt{x - 2} + 2$



2. $y = 2\sqrt[3]{x - 1} - 2$



Example 3 Finding Domain and Range

State the domain and range of the function in (a) Example 1 and (b) Example 2.

Solution

a. From the graph of $y = 2\sqrt{x + 1} + 1$ in Example 1, you can see that the domain is $x \geq -1$ and the range is $y \geq 1$.

b. From the graph of $y = -\sqrt[3]{x + 2} - 1$ in Example 2, you can see that the domain and range are both all real numbers.

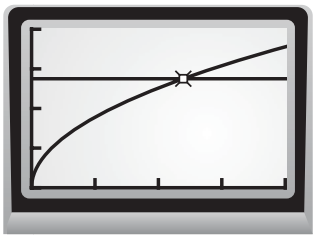
Your Notes

Example 4 ***Modeling with a Square Root Function***

The equation of the radius r of a circle in terms of the area A is $r = \sqrt{\frac{A}{\pi}}$. Use a graphing calculator to graph the model. Then use the graph to estimate the area of a circle that has a radius of 2.76 units.

Solution

Graph $y = \sqrt{\frac{x}{\pi}}$ and $y = 2.76$.



Choose a viewing window that shows the point where the graphs intersect. Then use the *Intersect* feature to find the x-coordinate of that point. You get $x \approx 23.93$.

The area is about 23.93 square units.

✔ ***Checkpoint*** **Complete the following exercises.**

3. State the domain and range of the function in (a) Checkpoint 1 and (b) Checkpoint 2.

(a) domain: $x \geq 2$; range $y \geq 2$

(b) domain and range: all real numbers

4. Graph the equation $y = -1.3\sqrt{x}$ on a graphing calculator. Then use the graph to estimate the value of x when y is -2.3.

about 3.13

Homework