

TSFX MASTER CLASSES 2020 UNIT 4 FURTHER MATHEMATICS

UNIT 3 & 4 WRITTEN EXAMINATION 2

Reading Time: 15 minutes **Writing Time:** 1 hour 30 minutes

QUESTION AND ANSWER BOOK

Student Name:

Structure of Book

Section A – Core	Number of questions	Number of questions to be answered	Number of marks
	9	9	36
Section B – Modules	Number of modules	Number of modules to be answered	Number of marks
	4	2	24
			Total 60

- Students are to write in blue or black pen.
- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials Supplied

- Question and answer book of 31 pages.
- Formula sheet.
- Working space is provided throughout the book.

Instructions

- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are **NOT** permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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SECTION A – Core

Instructions for Section A

Answer **all** questions in the spaces provided.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π , surds or fractions.

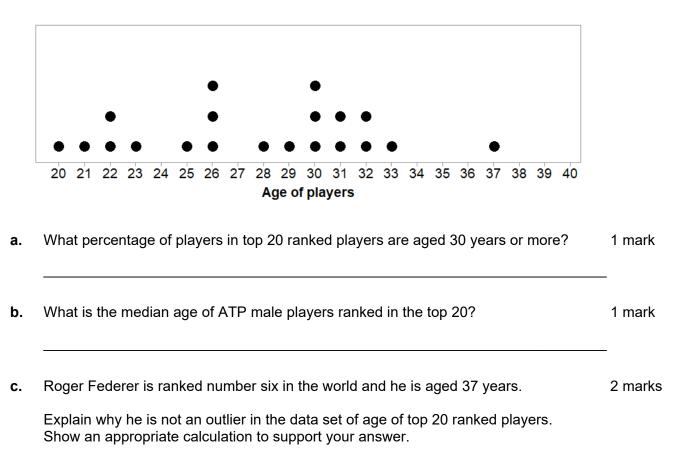
In 'Recursion and financial modelling', all answers should be rounded to the nearest cent unless otherwise instructed.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Data Analysis

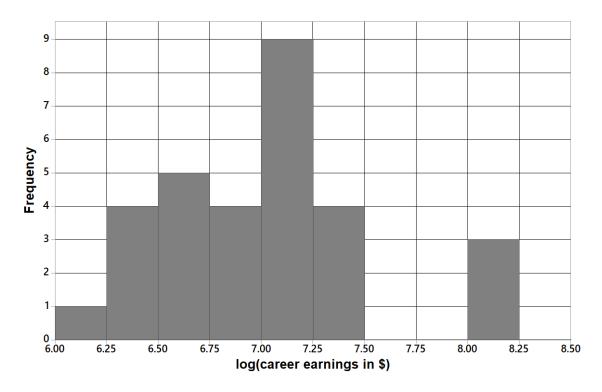
QUESTION 1 (4 marks)

The ages of the top 20 ranked male players in the Association of Tennis Professionals (ATP) is shown on the dot plot below:



QUESTION 2 (3 marks)

A histogram is shown below of the log_{10} (career earnings in dollars) for the top 30 ranked male players in the ATP.



a. The modal class interval for this data is between two career earning values. State 2 marks these two career earnings values, each correct to one significant figure.

b. Stefanos Tsitsipas has career earnings of \$3 633 180. Place a cross (X) in the bar on 1 mark the histogram that would contain Stefanos Tsitsipas' earnings.

ANSWER ON GRAPH

QUESTION 3 (3 marks)

The *average percentage of first serves in* for the top 100 ranked male tennis players in the ATP is normally distributed with a mean of 61.2% of serves and a standard deviation of 3.6% of serves.

a. What percentage of the top 100 ranked male players would average more than 72% 1 mark of first serves in?

The average percentage of first serves in for the top 100 ranked female tennis players in the Women's Tennis Association (WTA) is also normally distributed with a mean of 61.1% of serves and a standard deviation of 4.0% of serves.

b. Marin Cilic is a male tennis player with an average percentage of first serves in 2 marks of 59.0%.

Naomi Osaka is a female tennis player who also has an average percentage of first serves in of 59.0%.

Taking into account the statistics supplied, which player would be considered to be better at getting first serves in, in comparison to other players in their association?

Show calculations to support your answer.

QUESTION 4 (3 marks)

Winning a Grand Slam tournament in tennis is highly regarded. Only 149 male tennis players have won a Grand Slam tournament in the history of tennis, some of them many times.

The two way percentaged frequency table below shows the percentages, correct to the nearest whole percentage, of male players who have won at least one Grand Slam in the Open Era (post 1968) and before the Open Era (1968 and before).

The players are categorised by the total number of Grand Slam wins that they had.

	Before the Open Era	Open Era
5 or less Grand Slam wins	85%	75%
More than 5 to less than 10 Grand Slam wins	12%	15%
More than 10 Grand Slam wins	3%	10%
Total	100%	100%

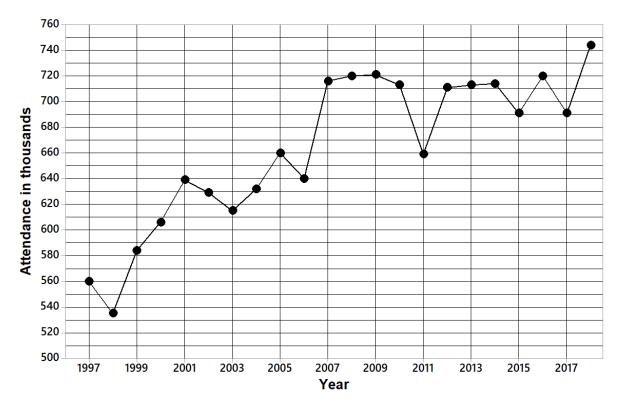
- **a.** Which of the variables *Era played* or *Total Grand Slams won* is the explanatory 1 mark variable?
- b. Does the information in the two way percentaged frequency table support the 2 n contention that the number of Grand Slam tournaments won is associated with the era in which the player plays?

Explain your answer with reference to appropriate percentages.

2 marks

QUESTION 5 (7 marks)

The graph below shows the *Attendance (in thousands)* at the Australian Open each year from 1997 to 2018.



a. Describe the trend in *Attendance (in thousands)* during the years shown in the graph 1 mark above.

The least squares regression equation, correct to three significant figures, for the relationship between *Attendance (in thousands)* and *year* is given by:

Attendance (in thousands) = $-15\ 100 + 7.84 \times year$

The correlation coefficient for this relationship is r = 0.865 correct to 3 decimal places.

b. Add the least squares regression line to the time series plot above.

1 mark

ANSWER ON GRAPH

c. State the percentage of the variation in attendance in thousands that can be explained 1 mark by the variation in the year. Give your answer correct to the nearest whole percent.

The *Attendance (in thousands)* at the Wimbledon Open over the same years is given in the table below:

Year	Attendance (in thousands)
1997	382
1998	402
1999	410
2000	440
2001	432
2002	481
2003	469
2004	469
2005	415
2006	465
2007	433
2008	431
2009	472
2010	505
2011	494
2012	495
2013	493
2014	491
2015	453
2016	494
2017	473
2018	473

d. Determine the equation of the least squares regression line that would predict the 2 marks *Attendance (in thousands)* from the *year*.

Write the gradient and intercept values correct to three significant figures.

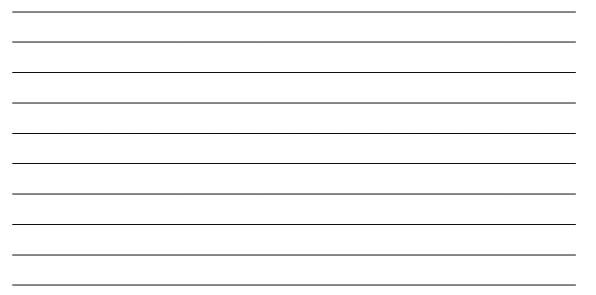
Attendance (in thousands) =

+

× year

Since 1997, the equations of the least squares lines for the Australian Open and Wimbledon Open have predicted that the attendance at the Australian Open will always exceed the attendance at the Wimbledon Open.

e. Explain why, quoting the values of appropriate statistics. 2 marks



QUESTION 6 (4 marks)

A sporting goods supplier notices that sales of tennis racquets are seasonal. The seasonal indices for racquet sales for Summer and Winter are shown below:

Season	Summer	Autumn	Winter	Spring
Seasonal Index	1.62		0.43	

On average twice as many racquets are sold in Spring as are sold in Autumn.

a. Complete the two missing seasonal indices to the table above.

ANSWER IN TABLE

The seaonal index for racquet sales for this supplier in Winter is 0.43.

b. What does this seasonal index indicate about the average racquet sales in Winter for 1 mark this supplier?

The deseasonalised racquet sales can be predicted using the least squares regression line

Deseasonalised sales = 103.12 + 6.34 × time period

where Summer 2018 = time period 1, Autumn 2018 = time period 2, etc.

c. During Summer one year, the actual racquet sales first reaches 300 racquets. 2 marks

Correct to the nearest whole time period, in which year does the least squares line predict that will happen for the first time?

1 mark

Recursion and Financial Modelling

QUESTION 7 (5 marks)

Tom wants to buy a new computer.

Twelve months ago he invested \$4000 in an account that was paid 6.9% per annum interest compounding monthly.

- **a.** What was the monthly rate of interest paid into Tom's account? 1 mark
- **b.** Write a recurrence relation, in terms of T_{n+1} and T_n , that gives the balance of Tom's 1 mark account *n* months after he started the investment.

Two months after Tom invested his money, his account balance was \$4046.13, correct to the nearest cent.

c. Using recurrence, write down the calculations that would show that Tom's account 2 marks balance is \$4046.13 after two months.

d. How much money does Tom have to buy his computer now that he has had the 1 mark account for 12 months?

QUESTION 8 (4 marks)

Tom has a printer that he uses in his work. He purchased the printer for \$250.

It will depreciate by \$1.25 for every 500 pages that are printed.

What is the book value of the printer after it has printed 15 000 pages? 1 mark a. b. Tom will write off the printer after a four year period when the book value is zero. 1 mark How many pages will Tom print on average per year? Tom could have depreciated his printer using reducing balance depreciation. C. 2 marks The rate of reducing balance depreciation would have resulted in the same book value after three years as the value from unit cost depreciation. Determine the reducing balance depreciation rate that would achieve this result. Give your answer correct to the nearest whole percent.

QUESTION 9 (3 marks)

Tom's business has been going well and he has saved \$45 000 in his superannuation account. His superannuation account is paying 7.5% per annum compounding monthly.

He is currently 30 years old and he wants to retire in 20 years time.

a. If Tom wants a total of \$700 000 when he retires, how much would he need to 1 mark contribute every month to achieve his aim?

Tom has a retirement plan with his \$700 000 balance. His retirement account will pay 6.5% per annum compounding monthly.

He will take \$4000 per month for ten years, then \$5000 per month for the next ten years and then increase his payment to \$6000 per month until his balance reduces so that the account can no longer pay \$6000. He will then take a final smaller payment.

b. How long would Tom's retirement account last, in total, before the account balance is 2 marks reduced to zero?

END OF SECTION A

SECTION B – Modules

Instructions for Section B

Select two modules and answer all questions within the selected modules.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π , surds or fractions.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

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Module 1: Matrices

QUESTION 1 (3 marks)

A local tennis club runs competition on both Wednesdays (W) and Saturdays (S). The competition has games for both adults (A) and children (C).

The numbers of adults and children, who play on Wednesdays and Saturdays, is shown in the matrix, R, below:

$$\begin{array}{ccc}
A & C \\
R = \begin{bmatrix} 100 & 10 \\
120 & 150 \end{bmatrix} & W \\
S & S \\
\end{array}$$

a. How many adults play on Saturdays?

Adults pay \$6.50 per game and children pay \$2.50 per game. The total money collected from adults and children on each Wednesday and Saturday is displayed in a 2×2 matrix, *M*.

b. Write a matrix, G, such that $R \times G = M$.

The number of people playing at the club changes the next year. The new numbers of people playing is obtained from the following matrix product:

$$\boldsymbol{E} \times \boldsymbol{R} = \begin{bmatrix} 1.1 & 0\\ 0 & 0.8 \end{bmatrix} \times \boldsymbol{R}$$

c. Describe the effect of pre-multiplying **any** 2×2 matrix by matrix *E*.

1 mark

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1 mark

1 mark

QUESTION 2 (4 marks)

Amara (A), Bhavana (B), Chelsea (C), Danika (D), Etta (E) and Fatima (F) all play at the tennis club. One week they decide to have a round-robin tournament and they use the permutation matrix multiplication below to determine the playing schedule in round one:

0 0	0 1		1 0	0 0	0 0		A B C D E F	
0	0	1	0	0	0 1		C	
0	0	0	0 0	0 0	1		D	
1	0	0	0	0	0		E	
0	0	0	0	1	0_		F	

This matrix product will determine who each person plays against. From the product matrix row 1 will play row 2, row 3 will play row 4 and row 5 will play row 6.

a. List who each player will play against in the first round according to this matrix. 1 mark

The players complete a number of rounds of tennis so that each person has played each other person once. The winner of each game is shown in the adjacency matrix below. A win is represented by a "1" in the winner's row and the loser's column:

					los	ser			
			Α	В	С	D	Ε	F	
		Α	[0]	1	1	1	0	1]	
D =	winner	В	0	0	1	0	1	0	
		С	0	0	0	1	1	1	
D =	WIIIICI	D	0	1	0	0	1	0	
		Ε	1	0	0	0	0	1	
		F	0	1	0	1	0	0	

The placings of the tournament are determined using the sum of one-step and two-step dominances.

- **b. i.** Amara wins the tournament and Chelsea comes second. Who came third? 1 mark
 - **ii.** Explain the meaning of the element in the first row and fifth column of the matrix 2 marks that sums the one-step and two-step dominances.

QUESTION 3 (5 marks)

The adult tennis players, who play in the local competition between a number of clubs, play in a number of different divisions, A, B and C. Each week some players will not play, N.

The players transition between the three divisions and not playing according to the matrix T below:

this week						
			С			
т _	[0.8	0.1	0	0.2]	Α	
I =	0.1	0.7	0.2	0.2	В	maxt wool
	0	0.1	0.7	0.4	С	πέχι ωθέκ
	0.1	0.1	0.1	0.2	Ν	next week

At one of the clubs, in one particular week, 50 people played in A division, 40 played in B division, 30 played in C division and 20 people did not play.

a. Write down the calculation that shows that, at this club, 33 people will play in C 1 mark division the following week.

Another much larger club has 100 people playing at each of levels A, B and C in the fifth week (W_5) of competition. None of their players missed the fifth week.

b. Of the players at this club who played in Division C in the seventh week (W_7), what 2 marks percentage played in Division B in the sixth week (W_6)?

Give your answer correct to one decimal place.

c. A total of 1000 people play tennis in the larger competition.

1 mark

How many of these people, correct to the nearest whole person, will play in Division B in the long run?

Another club have 100 people playing in each of divisions A, B and C. They also have 100 people who do not play each week (N).

In order keep numbers at the same level each week they have the following rules:

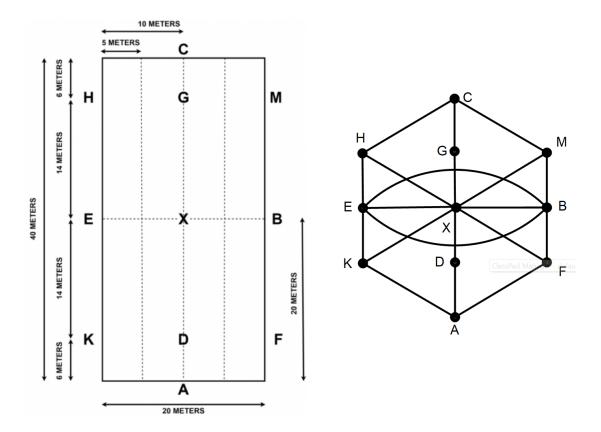
- Each week 50% of those in each division or not playing will stay in that division or not play the following week.
- The other 50% of people in each divison or not playing must move to one of the other divisions or not play the following week.
- The people who move do not need to all go to the same division or play the following week.
- **d.** Complete the transition matrix below that would ensure that there are always 100 1 mark people in each division or not playing each week, given these rules.

$$\begin{array}{cccc}
this week \\
A & B & C & N \\
\begin{bmatrix} & & & \\$$

Module 2: Networks and decision mathematics

QUESTION 1 (2 marks)

Luca is setting up a small dressage arena to practise his riding skills at his parent's property. A diagram of the arena along with the letters used to denote locations within and around the arena are shown below. The associated network of allowed routes is also shown:



After fencing the arena, Luca starts a basic training drill by riding into the arena at A and then passing through D, X, G, C, M, B, E and B.

Each of the letters in the arena could be considered a vertex in a network and the allowed routes within th dressage arena are edges.

a. Explain why the route ADXGCMBEB is not considered to be a **path** in network. 1 mark

1 mark

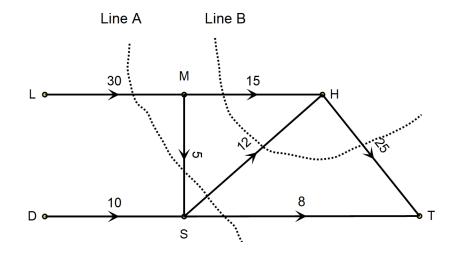
b. Explain why the network is considered to be a planar network.

QUESTION 2 (5 marks)

Luca sets up a series of pipes from a large dam (L) and a small dam (D) at his parent's property to fill a water tank (T) for the horses. The pipes also need to go to and from the house (H), the machinery shed (M) and the shearing shed (S).

A diagram of the flow network for these pipes, including the capacity of each pipe, in L/min, is shown in the diagram below.

Two lines, Line A and Line B have also been added to the flow network.

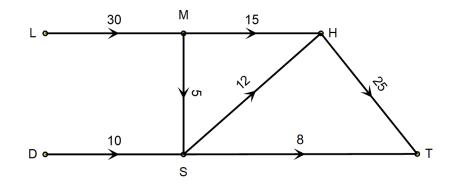


- a. Line B was added by a student who was attempting to draw a "cut" through the 1 mark network. Explain why Line B is **not** a suitable cut to determine network flow.
- **b.** State the capacity of the cut made by Line A, in L/min.

1 mark

c. State the maximum flow from the two sources L and D, to the sink T for this network, 1 mark in L/min.

d. Add the line that would represent the minimum cut to the diagram below.



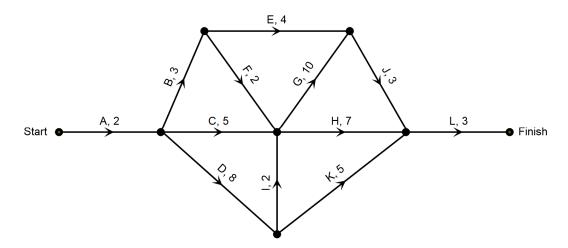
There is insufficient flow to the water tanks (T) for the horses. Luca will replace the pipe from the machinery shed (M) to the shearing shed (S) with a new pipe of capacity 20 L/min.

e. What is the new maximum flow to the water tanks (T) if this pipe is replaced? 1 mark

1 mark

QUESTION 3 (5 marks)

To set up Luca's dressage arena a number of tasks had to be completed. The tasks are shown in the activity network below, along with the duration in days of each activity:



- a. State the activities that have three immediate predecessors.
- **b.** What is the minimum time in which Luca can complete his dressage arena project? 1 mark
- c. Two activities have a latest start time of 7 days. State these two acitivities. 1 mark

In order to be ready for a local competition, Luca wants the dressage arena project to take no longer than 3 weeks (21 days).

He can pay an extra \$200 per day of reduction to reduce the duration of any activity, other than Activity A or Activity L. No activity can be reduced to less than two days' duration.

d. How much will it cost Luca to reduce the completion time of his project to 21 days? 2 marks

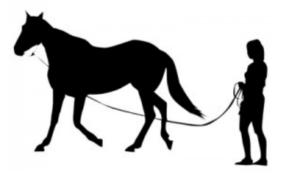
1 mark

Module 3: Geometry and Measurement

QUESTION 1 (2 marks)

Amber owns a horse that she uses in dressage competitions.

As part of Amber's horse's training she "lunges" the horse on a long rope. This means that she stands in the centre of an arena and the horse moves in a circle around her, with the length of the lunge rope forming the radius of the circle.



a. Amber starts with a 10 m long lunge rope and the horse walks around her. 1 mark

What distance will the horse walk for each circle? Give your answer in metres correct to one decimal place.

b. Amber has a circular lunging yard with an area of 1018 m².

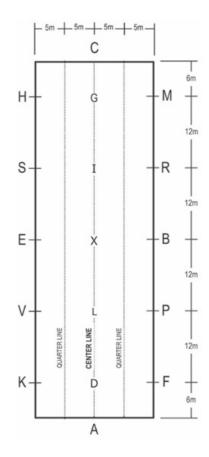
1 mark

Given that the horse must stay at least one metre away from fence and Amber stands in the centre of the lunging yard, what is the longest length of lunge rope Amber can use? Give your answer in metres, correct to one decimal place.

QUESTION 2 (3 marks)

Amber rides her horse in dressage competitions. A diagram of a 60 m by 20 m dressage arena is shown:

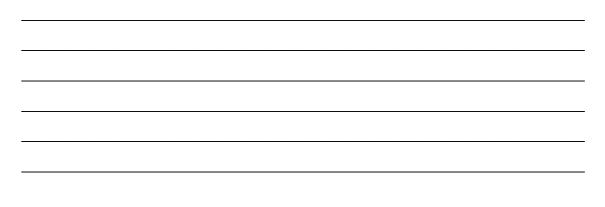
One part of Amber's dressage test is a "half pass" forming a straight line from K to X, followed by a circle passing through both X and B.



a. Show that, if Amber performs these moves correctly, she will have travelled 57.4 m, 1 mark correct to one decimal place.

b. The dressage arena has a rectangular angled colorbond roof with support posts at K, 2 marks E, H and F, B, M. The support posts at K, E and H are two metres longer than the support posts at F, B and M, so that the roof is two metres higher along that side.

Given that the 60 m by 20 m arena is completely covered, what is the total area of colorbond used in this roof? Give your answer in metres squared, correct to the nearest metre squared.



QUESTION 3 (4 marks)

Amber lives in Adelaide (35°S, 139°E). She flies to Fuji, Japan, (35°N, 139°E) to attend a dressage competition.

a. Assume that the radius of the Earth is 6400 km.

1 mark

What is the distance between Adelaide and Fuji, in kilometres, correct to the nearest kilometre?

Before she leaves for Japan, Amber rings her friend, Renato, who lives in Buenos Aires, Argentina (35°S, 58°W). She makes the phone call at 7 pm on a Friday, Adelaide time.

b. Given that each 15° difference in longitude represents a time difference of one hour, 1 mark what was the time in Buenos Aires when Amber rang Renato?

Give your answer correct to the nearest whole hour.

c. Amber and Renato are sad that they live so far apart. What is the shortest distance 2 marks from Adelaide to Buenos Aires, if Amber and Renato want to visit each other?

Give your answer correct to the nearest kilometre.

QUESTION 4 (3 marks)

Amber feeds her horses using an automatic feeder that is shaped as a hexagonal based pyramid.

The feeder holds 12.25 m³ of food for the horses and stands 3 m high.

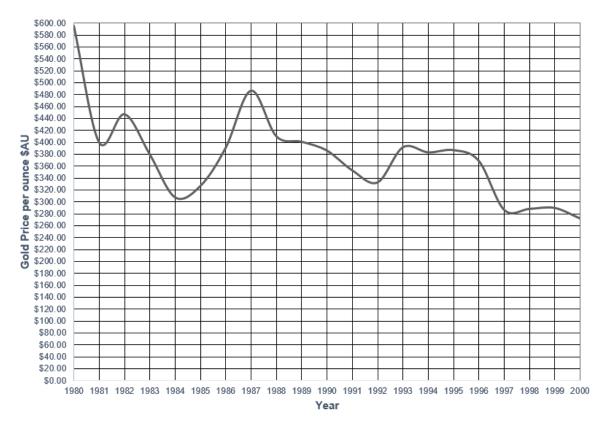
- **a.** What is the area of ground, in m², that is covered by the feeder's base? 1 mark
- **b.** The feeder was initially full, but after two days the horses had consumed enough food 2 marks that the height of food remaining in the feeder is now 2 metres.

What fraction of the food initially in the feeder remains?

Module 4: Graphs and Relations

QUESTION 1 (2 marks)

The graph below shows the price per ounce of gold, in Australian dollars, during the years 1980 through to 2000. Each line on the horizontal scale represents the 1st of January of the listed year, so the graph starts at the 1st of January, 1980, and finishes on the 1st of January, 2000.



a. During how many calendar years did the price of gold drop by \$60 per ounce or more? 1 mark

Brigid is a prospector who found a small nugget of gold on the 1st of January, 1981.

b. What was the average loss in dollars per ounce per year, if she sold her nugget on the 1 mark 1st of January, 1997?

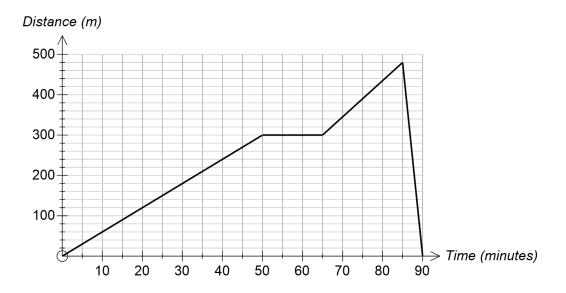
Give your answer correct to the nearest dollar per ounce per year.

QUESTION 2 (4 marks)

Brigid uses a metal detector to find gold. She has a favourite place to search for gold that is alongside a straight road.

One day Brigid parks her car and walks slowly from a car park along the road using her detector. She stops at a location where she gets a signal from the detector for a while, but then continues slowly along the road. Eventually the battery on her detector goes flat, so she turns and walks straight back to her car.

The distance-time graph below shows her progress, in metres per minute, where the distance in metres is the distance she is in a straight line from her car.



a. What was Brigid's average speed in metres per minute during the first 50 minutes of 1 mark her journey?

The equation of the line that gives the *distance*, in metres, that Brigid is from her car, after t minutes, is of the form $distance = a \times t + b$.

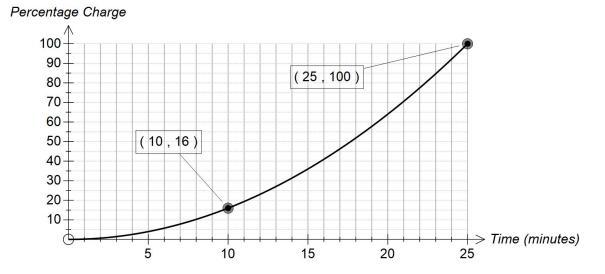
b. Give the values of *a* and *b* for the equation of the line segment between 65 minutes 1 mark and 85 minutes after she leaves her car.

c. How far, in metres, is Brigid from her car 88 minutes after she starts her journey? 2 marks

QUESTION 3 (4 marks)

After using her metal detector, Brigid will need to recharge the battery. The metal detector has a display that indicates the percentage charge from 0% to 100%.

Brigid records that her metal detector charges according to the graph below:



a. Write the relationship between the percentage charge, *C*, and the number of minutes, 1 mark *t*, for the metal detector.

Brigid's charger stops working and she buys a new charger. She determines that this charger charges the metal detector according to the equation below:

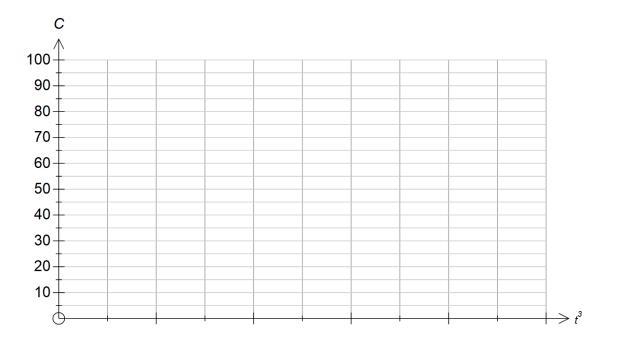
$$C = \frac{1}{270} \times t^3$$

b. How long will it take for the metal detector to reach 100% charge?

1 mark

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c. Add the graph showing the relationship between percentage charge (C) and t³ for 2 marks the new charger on the axes below. Include a suitable scale on the horizontal axis and label at least one point.



QUESTION 4 (2 marks)

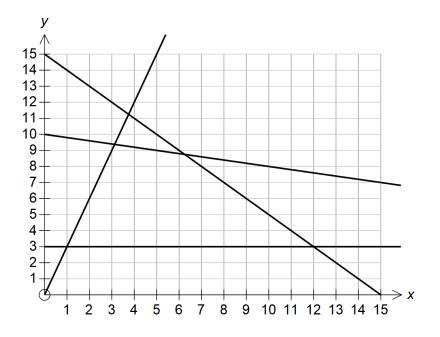
Although Brigid loves prospecting, she does not always make enough money to pay all her bills, so she lectures and runs tutorials on metallurgy at a local university. Each tutorial and lecture runs for exactly one hour.

For this problem, x is the number of hours of tutorials and y is the number of hours of lectures that Brigid completes.

The following inequalities apply to Brigid's university work:

 $y \ge 3$ $x + y \le 15$ $y \le 3x$ $10x + 50y \le 500$

The graph below shows the boundary lines that form the feasible region for this linear programming problem:



Brigid is paid \$40 per hour long tutorial and \$250 per hour long lecture. 2 marks

What is the maximum income that Brigid can earn each week from tutorials and lectures?

END OF QUESTION AND ANSWER BOOK