UNIT 3 MATHS METHODS
PROBABILITY NOTES FOR THE VCAA EXAMS

WRITTEN BY A STUDENT WHO OBTAINED A NEAR PERFECT STUDY SCORE

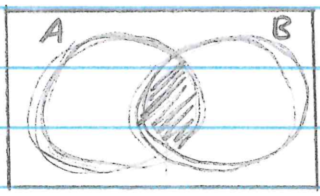
PROBABILITY

Discrete random variables and probability distributions:

Basic probability:

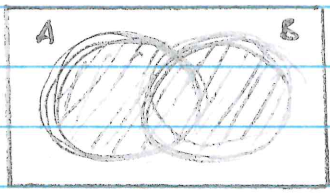
Consider the random experiment of rolling a fair, 6-sided die.

- TRIAL is the number of times an experiment is performed
- SAMPLE SPACE = $\mathcal{E}$ = set of all possible outcomes = $\{1,2,3,4,5,6\}$
- OUTCOME/EVENT is the result of an experiment
  e.g. A = the event "rolling an even number" $A = \{2,4,6\}$
- THE INTERSECTION EVENT $A \cap B$
  $A \cap B$ is said to have occured if an outcome that is in both A and B has occured



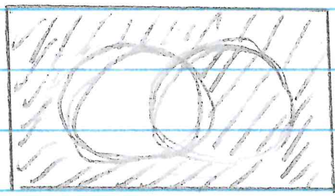
* $A \cap B$ is the same as $B \cap A$
* If $A \cap B$ are mutally exclusive events then $A \cap B = \emptyset$ ← empty set

- THE UNION EVENT $A \cup B$
  $A \cup B$ is said to have occured if an outcome that is in either A or B has occured.



* $A \cup B$ is the same as $B \cup A$
* $P_r(A \cup B) = P_r(A) + P_r(B) - P_r(A \cap B)$

- THE COMPLEMENT EVENT A'
  A' is said to have occured if an outcome that is not in A has occured



* $P_r(A') = 1 - P_r(A)$

- MUTALLY EXCLUSIVE OR DISJOINT SETS are sets that have no element in common
  $P_r(A \cap B) = 0$
- $P_r(A' \cap B) + P_r(A \cap B) = P_r(B)$
- $P_r(A' | B) = 1 - P_r(A | B)$
- $P_r(A) = P_r(A \cap B) + P_r(A \cap B')$
- If M is a subset of K, then $P_r(K \cup M) = P_r(K)$

- $P_r(A' \cap B') = 1 - P_r(A \cup B)$

Rules for probability:

Probability measures the likelihood of an outcome or event. It is measured on a scale of 0 (impossible) to 1 (certain). The rules are:

1. $0 \le Pr(A) \le 1$
2. Probabilities for all outcomes for an event add to 1
3. $Pr(\emptyset) = 0$
4. $Pr(A') = 1 - Pr(A)$
5. $Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B)$
6. In a fair trial where each outcome is equally likely.

$$Pr(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

NOTE: 0 and 1 are valid probabilities so don't cross them out when your answer say one/zero

Probability tables (2 way table)

These are used to help solve problems with only two events (A and B)

	A	A'	
B	$Pr(A \cap B)$	$Pr(A' \cap B)$	$Pr(A')$
B'	$Pr(A \cap B')$	$Pr(A' \cap B')$	$Pr(B')$
	$Pr(A)$	$Pr(A')$	1

example:

1. A new drug has some side-effects. 8% of patients suffer headaches but not nausea and 12% suffer nausea but not headaches. If 75% of patients have no side effects, what percentage suffer from both?

	H	H'	
N	5%	12%	17%
N'	8%	75%	83%
	13%	87%	100%

$\Rightarrow Pr(H \cap N) = 5\%$

Conditional Probability

Pr (A|B) is the probability that event A occurs when it is given (or known) that B has occurred.

Rules:

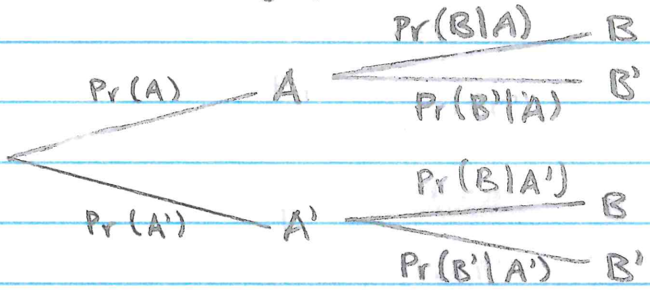
1. $Pr(A|B) = \dfrac{Pr(A \cap B)}{Pr(B)}$

2. $Pr(A \cap B) = Pr(A|B) \times Pr(B)$

3. If A and B are independent, $Pr(A|B) = Pr(A)$ and $Pr(A \cap B) = Pr(A) \times Pr(B)$

Tree diagram

These are a very useful problem solving tool - especially for conditional probability problems.

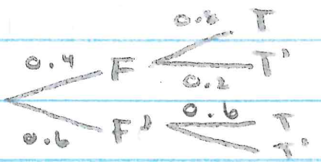


examples:

① There is a daily flight from Paradise Island to Melbourne. The probability of the flight departing on time, given that there is fine weather on the island is 0.8, and the probability of the flight departing on time given that the weather on the island is not fine is 0.6.

In March the probability that the weather is fine is 0.4

a) The flight from Paradise island departs on time?



$\Rightarrow Pr(T) = 0.4 \times 0.8 + 0.6 \times 0.6$

$= 0.32 + 0.36$

$= 0.68$

b) The weather is fine on the island, given that the flight is on time.

$Pr(F|T) = \dfrac{0.32}{0.68} = \dfrac{8}{17}$

② Given that $Pr(A|B) = p$, $Pr(B) = p^2$ and $Pr(A) = p^{1/3}$.
Find the $Pr(B|A)$.

$$p = \frac{Pr(A \cap B)}{p^2} \qquad Pr(B|A) = \frac{p^3}{p^{1/3}}$$

$$Pr(A \cap B) = p^3 \qquad = p^{8/3}$$

Combinations and Permutations

The factorial is denoted $n!$ and is the product of all positive integers less than or equal to n.

e.g. $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

$0! = 1$ by definition.

→ Permutations

n objects, select r, order matters

$${}^nP_r = \frac{n!}{(n-r)!}$$

→ Combinations

n objects, select r, but order doesn't matter

$${}^nC_r = \binom{n}{r} = \frac{n!}{(n-r)! \times r!} = \binom{n}{n-r}$$

NOTE:

- $\binom{n}{1} = n = \binom{n}{n-1}$
- $\binom{n}{0} = 1 = \binom{n}{n}$

example:

① If six women and five men are interviewed for five positions
Find the probability that

a) $Pr(3M, 2W) = \frac{\binom{5}{3}\binom{6}{2}}{\binom{11}{5}} = \frac{\text{favourable}}{\text{possible}}$

$= \frac{25}{77}$

b) $Pr(W \geq 1) = 1 - Pr(W = 0)$

$= 1 - \frac{\binom{5}{5}\binom{6}{0}}{\binom{11}{5}}$

$= \frac{461}{462}$

c) $Pr(W = 2 | W \geq 1) = \frac{\binom{6}{2}\binom{5}{3}}{\binom{11}{5}} \div \frac{461}{462}$

$= \frac{25}{77} \div \frac{461}{462}$

$= 150/467$

example

① Two events A and B are such that $Pr(A) = 0.3$, $Pr(B) = 0.1$ and $Pr(A \cup B) = 0.37$. Are A and B independent

$$Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B)$$

$$0.37 = 0.3 + 0.1 - Pr(A \cap B)$$

$$Pr(A \cap B) = 0.03$$

If independent, $Pr(A) \times Pr(B) = Pr(A \cap B)$

$$0.3 \times 0.1 = 0.03$$

$\Rightarrow$ independent.

Déscrete random variables:

Discrete random variables are random variables that can only assume a countable number of values.

e.g. shoe size, number of children

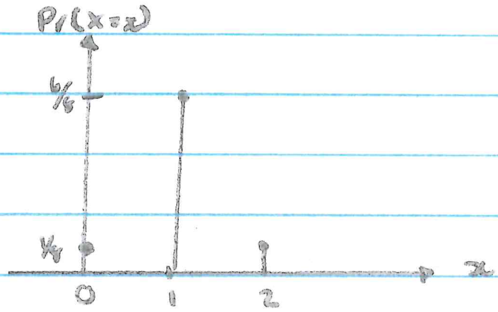
It can be any value within an interval

Probability distribution function: (PDF)

The Pdf shows the outcomes and associated probabilities. This may take the form of a table, graph or function. The notation $Pr(X=x)$ means the probability that random variable X is equal to the numerical value x.

E.g.

x	0	1	2
$Pr(X=x)$	$1/8$	$6/8$	$1/8$



$p(x) = \frac{1}{57}(18-3x), \; x \in \{-2, 1, 2, 4\}$

NOTE:

The general rules:

- $0 \leq Pr(X=x) \leq 1$
- The sum of the $Pr(X=x)$ values is 1