SEQUENCES AND SERIES

A "sequence" is a set of numbers or terms. A "series" is the sum of these numbers or terms.

$\{15, 20, 25, ...\}$ is a sequence.

$15 + 20 + 25 + ...$ is a series.

Every term of a sequence has a corresponding 'n' value, which gives the term's position in the sequence. Hence n is a counting number of 1, 2, 3, … (never zero, negative or fractional).

For example: $\{5, 14, 23, 32, ...\}$

$$\begin{array}{cccc} \nearrow & \uparrow & \uparrow & \nwarrow \\ n=1 & n=2 & n=3 & n=4 \end{array}$$

The "General Term, T_n" (sometimes written as U_n) is also called the "$n\ th$ term"

It is a formula which will predict any term in the sequence by simply replacing the n in the formula with the corresponding n value.

The "Sum of Terms, S_n" is a formula which will predict the sum of terms up to and including the $n\ th$ term.

WATCH OUT!
The use of T_n and S_n formulas is very similar to function notation.

QUESTION 1

(a) Given $T_n = 9n - 4$, find $T_1, T_2, T_3, T_k, T_{k+1}$

$T_1 = 9(1) - 4 = 5$

$T_2 = 9(2) - 4 = 14$

$T_3 = 9(3) - 4 = 23$

T_k = _________________________ = __________________________

T_{k+1} = _________________________ = __________________________ = ______________

(b) Which term equals 149?

Let $T_n = 149$

$\therefore 9n - 4 = 149$

$9n = 153$ $\quad\quad n = 17$ i.e. The $17th$ term is 149.

(c) Is 200 a term of this sequence?

Let $T_n = 200$

$\therefore 9n - 4 = 200$

$9n = 204$

$\therefore n = 22.7$

Hence 200 is not a term of this sequence since n cannot be a fraction.

<u>REMEMBER</u>

n must be a counting number.

(d) Find the first term to exceed 1000.

Let $T_n > 1000$

$\therefore 9n - 4 > 1000$

$9n >$ _____

$\therefore n >$ _____

The first positive integral value satisfying this is $n =$ _____ . Hence:

$T_n = 9n - 4$

$\therefore$ ___ $= 9($___$) - 4$

$=$ ____

The first term to exceed 1000 is ___________.

(e) Find S_3.

Here we are not given a formula for S_n, but we do know $T_1 =$ ___ $T_2 =$ ___ $T_3 =$ ___

Hence $S_3 = T_1 + T_2 + T_3$

$=$ ______ $+$ ______ $+$ ______

$=$ ______

(f) Show that the formula $S_n = \dfrac{9n^2}{2} + \dfrac{n}{2}$ also gives the correct sum for S_3.

$$S_3 = \frac{9(\ )^2}{2} + \frac{(\ )}{2}$$

$=$ ________

Which agrees with the answer in part (e) above.

QUESTION 2

(a) Find the first term and fifth term of the sequence with the general n^{th} term $U_n = 2.3^{n-3}$.

$U_1 = 2.3^{(1)-3} = 2.3^{-2} = \dfrac{2}{9}$

$U_5 = 2.3^{(\ )-3} = 2.\_\_\_\_\_ = \_\_\_\_\_\_\_$

(b) Is 13122 a term of this sequence? Is 1482?

Let $U_n = 13122$ | $3^6 = 729$

i.e. $2.3^{n-3} = 13122$ | $3^7 = 2187$

$3^{n-3} = \_\_\_\_\_$ | $3^8 = 6561$

$\therefore 3^{n-3} = 3\_\_\_\_\_$ By using the constant multiplying function on the calculator.

$n - 3 = \_\_\_\_\_$

$n = \_\_\_\_\_$

So yes, 13122 **is** a term of this sequence (the _____th term).

Let $U_n = 1482$

i.e. $2.3^{n-3} = 1482$

$3^{n-3} = \_\_\_\_\_$

But $3^6 = 729$ and $3^7 = 2187$.

$\therefore$ No positive integral value of n is possible.

$\therefore$ 1482 is not a term of this sequence.

QUESTION 3

Evaluate $\sum_{r=1}^{3}\left(2r^2-7r+5\right)$.

Solution

Here the general term is $T_r = 2r^2 - 7r + 5$

(r is being used instead of n … it makes no difference).

$T_1 = 2(1)^2 - 7(1) + 5 = 0$

$T_2 = 2(2)^2 - 7(2) + 5 = -1$

$T_3 = 2(3)^2 - 7(3) + 5 = 2$

$\therefore \sum_{r=1}^{3} T_r = 0 +$ ________ + ________

= ________

MORE ON THE SUM OF TERMS, S_n

$S_1 = T_1$

$S_2 = T_1 + T_2 \qquad = S_1 + T_2$

$S_3 = T_1 + T_2 + T_3 \qquad = S_2 + T_3$

$$S_n = \underbrace{T_1 + T_2 + \ldots + T_{n-1}}_{S_{n-1}} + T_n$$

$S_n = S_{n-1} + T_n \quad (n > 1)$

$S_n - S_{n-1} = T_n$

$$\boxed{\begin{array}{c} T_n = S_n - S_{n-1},\ n > 1 \\ T_1 = S_1,\ n = 1 \end{array}}$$