tsfx THE SCHOOL FOR EXCELLENCE

HOW WELL DO YOU KNOW YOUR COURSE MATERIALS?

These questions (and many others) will be addressed in detail in the TSFX "Unit 3 Exam Revision Lectures" in September & October 2020.

UNIT 3 MATHS METHODS

Try the following VCAA questions to see how well your learning is translating into examination applications.

Question 1

The graph of $y = \tan(ax)$, where $a \in R^+$, has a vertical asymptote $x = 3\pi$ and has exactly one x-intercept in the region $(0, 3\pi)$.

The value of a is

A. $\frac{1}{6}$

B. $\frac{1}{3}$

C. $\frac{1}{2}$

D. 1

E. 2

Question 2

Consider the functions $f : R^+ \to R,\ f(x) = x^{\frac{p}{q}}$ and $g : R^+ \to R,\ g(x) = x^{\frac{m}{n}}$, where p, q, m and n are positive integers, and $\frac{p}{q}$ and $\frac{m}{n}$ are fractions in simplest form.

If $\{x : f(x) > g(x)\} = (0, 1)$ and $\{x : g(x) > f(x)\} = (1, \infty)$, which of the following must be **false**?

A. $q > n$ and $p = m$

B. $m > p$ and $q = n$

C. $pn < qm$

D. $f'(c) = g'(c)$ for some $c \in (0, 1)$

E. $f'(d) = g'(d)$ for some $d \in (1, \infty)$

Question 3

The equation $(p-1)x^2 + 4x = 5 - p$ has no real roots when

A. $p^2 - 6p + 6 < 0$

B. $p^2 - 6p + 1 > 0$

C. $p^2 - 6p - 6 < 0$

D. $p^2 - 6p + 1 < 0$

E. $p^2 - 6p + 6 > 0$

Question 4

The sum of the solutions of $\sin(2x) = \frac{\sqrt{3}}{2}$ over the interval $[-\pi, d]$ is $-\pi$.
The value of d could be

A. 0

B. $\frac{\pi}{6}$

C. $\frac{3\pi}{4}$

D. $\frac{7\pi}{6}$

E. $\frac{3\pi}{2}$

Question 5

The function f has the property $f(x) - f(y) = (y - x)f(xy)$ for all non-zero real numbers x and y.
Which one of the following is a possible rule for the function?

A. $f(x) = x^2$

B. $f(x) = x^2 + x^4$

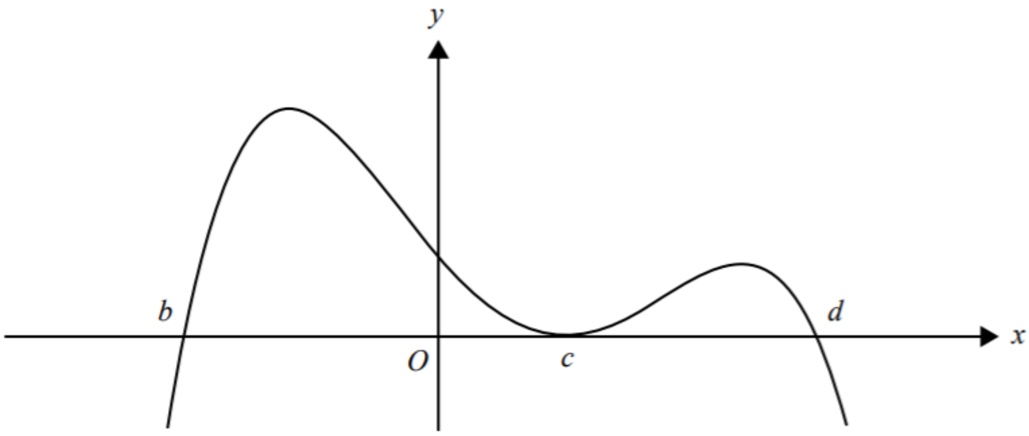
C. $f(x) = x\log_e(x)$

D. $f(x) = \frac{1}{x}$

E. $f(x) = \frac{1}{x^2}$

VIC: Level 3, 99 William Street, Melbourne VIC 3000
Phone: (03) 9663 3311 Fax: (03) 9663 3939
www.tsfx.com.au
admin@tsfx.com.au
NSW: PO Box r1407, Royal Exchange, Sydney NSW 1225
Phone: 1300 364 173 Fax: 1300 364 065

Question 6



The rule for a function with the graph above could be

A. $y = -2(x+b)(x-c)^2(x-d)$

B. $y = 2(x+b)(x-c)^2(x-d)$

C. $y = -2(x-b)(x-c)^2(x-d)$

D. $y = 2(x-b)(x-c)(x-d)$

E. $y = -2(x-b)(x+c)^2(x+d)$

Question 7

The transformation that maps the graph of $y = \sqrt{8x^3 + 1}$ onto the graph of $y = \sqrt{x^3 + 1}$ is a

A. dilation by a factor of 2 from the y-axis.

B. dilation by a factor of 2 from the x-axis.

C. dilation by a factor of $\frac{1}{2}$ from the x-axis.

D. dilation by a factor of 8 from the y-axis.

E. dilation by a factor of $\frac{1}{2}$ from the y-axis.

Question 8

The graphs of $y = mx + c$ and $y = ax^2$ will have no points of intersection for all values of m, c and a such that

A. $a > 0$ and $c > 0$

B. $a > 0$ and $c < 0$

C. $a > 0$ and $c > -\dfrac{m^2}{4a}$

D. $a < 0$ and $c > -\dfrac{m^2}{4a}$

E. $m > 0$ and $c > 0$

Question 9

Let $f : R \to R$ be a differentiable function. Then for all $x \in R$, the derivative of $f(\sin(4x))$ with respect to x is equal to

A $4\cos(4x)f'(x)$

B $\sin(4x)f'(x)$

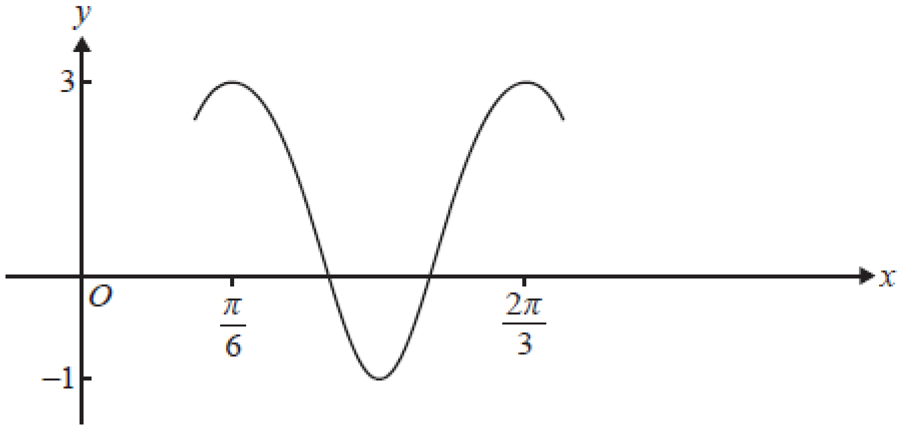
C $f'(\sin(4x))$

D $4f'(\sin(4x))$

E $4\cos(4x)f'(\sin(4x))$

How about the following VCAA-style questions?

Question 10



If $\cos(x) = f(x)$ then the graph shown could have equation

A $y = 2f\left(\dfrac{12x - 2\pi}{3}\right) + 1$

B $y = 2f\left(4\left(x - \dfrac{\pi}{6}\right)\right) - 1$

C $y = 2f\left(4\left(x - \dfrac{\pi}{6}\right) + 1\right)$

D $y = f\left(\dfrac{\pi}{2}\left(x - \dfrac{\pi}{6}\right) - 1\right)$

E $y = f\dfrac{\pi}{2}\left(x - \dfrac{\pi}{6}\right) - 1$

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Question 11

The function with rule $f(x) = \begin{cases} (x-a)^3 + 2, & x \le 0 \\ bx + \cos x, & x > 0 \end{cases}$ is differentiable for all values of x if

A $\quad a = 0$ and $b = 1$

B $\quad a = 1$ and $b = -3$

C $\quad a = 1$ and $b = 3$

D $\quad a = -1$ and $b = -3$

E $\quad a = -1$ and $b = 3$

Question 12

$\sin\theta = \sin\left(\frac{5\pi}{7}\right)$, $\theta \in R$ is equal to

A $\quad \theta = \pi k + (-1)^k \frac{5\pi}{7}$

B $\quad \theta = \pi k + (-1)^k \frac{2\pi}{7}$

C $\quad \theta = 2\pi k + (-1)^k \frac{2\pi}{7}$

D $\quad \theta = 2\pi k + (-1)^k \frac{5\pi}{7}$

E $\quad \theta = 2\pi k - (-1)^k \frac{5\pi}{7}$

Question 13

Could you find the range of $f(x) = \sin(\log_e x)$ in Exam 1 (no technology)? The answer is not $(-1, 1)$.

Question 14

Solve for x given that $\sqrt{x+2} = x - 10$, showing all working (no CAS allowed).

Did you get $x = 7, 14$ as your answer? If yes, then you would have lost the answer mark. Would you make this mistake in the exams?

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ANSWERS

Question 1 Answer is C
Question 2 Answer is E
Question 3 Answer is B
Question 4 Answer is C
Question 5 Answer is D
Question 6 Answer is C
Question 7 Answer is A
Question 8 Answer is D
Question 9 Answer is E
Question 10 Answer is A
Question 11 Answer is C
Question 12 Answer is B
Question 13 Answer is $[-1, 1]$
Question 14 Answer is 14

These questions (and many others) will be addressed in detail in the TSFX "Unit 3 Exam Revision Lectures" in September & October 2020

PTO FOR MORE EXAMPLES

APPLICATIONS

A student was asked to prove that the stationary point on the graph of $f(x)=\dfrac{4x}{(x-1)^2}$ is a minimum stationary point.

This student found the derivative $f'(x)=\dfrac{-4(x+1)}{(x-1)^3}$, let this equation equal to zero, and then found the corresponding value of x, which is -1.

The student then constructed the following table to prove the nature of the stationary point.

x	-2	-1	1
$f'(x)$	$-ve$	0	$+ve$

Why will the student be penalised for using $x=1$ to test the sign of the derivative?
Even if the student had used the correct values, they wouldn't receive any marks for the above response. What has the student failed to provide in order to obtain full marks?

The answer to the below question is option C.
Would you get this question correct in the exams?

Given that $f: R^+ \cup \{0\} \to R$ where $f(x)=6\sqrt{x}-x-5$ and $g: R \to R$ where $g(x)=x^2$, $f(g(x))$ is equal to:

A $(6\sqrt{x}-x-5)^2$

B $6x-x^2-5$

C $\begin{cases} 6x-x^2-5 & for\ x \ge 0 \\ -6x-x^2-5 & for\ x<0 \end{cases}$

D $\left(6\sqrt{x}-x-5\right)x^2$

E $f(g(x))$ does not exist

How well can you interpret graphs and equations?

For what values of p does $x^{\frac{3}{2}}=p+1$ exist? (The answer is $p \ge -1$).
Could you get this answer in the exams?

Given that: $f:[-1,1] \to R$ **where** $f(x)=\sqrt{1-x^2}$ **find** $g\left(f(x)\right)'(3)$**.**
$g: R \to R$ where $g(x)=2+x^2$

Most students would obtain -6 as their answer, which is in fact wrong.
Could you explain why $g\left(f(x)\right)'(3)$ does not exist in this case?

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