Introduction:

To catch the attention of customers, vacuum cleaner salesmen will sometimes reverse the air flow in a cleaner and then balance a beach ball in the exhaust jet. Two observations can be made: firstly, the ball does not fall out of the airstream. This occurs because the air from the jet curves around the ball equally on all sides, holding the ball in the path. Secondly, the ball will oscillate vertically in the airstream. This occurs because the upwards force of the air on the ball, which counterbalances the downwards force of gravity, constantly changes after the initial lift when first placed in the path of the airstream

Aim:

The aim of this investigation is to find the average height that spherical objects will hover over an upwards airstream coming out of an approximately 0.05 m circular opening. The dependent variables, height, amplitude of oscillation and period of oscillation, are to be investigated in relation to the independent variables, mass and ball radius. Balls of five varying radii and four varying masses are used, altogether giving 20 balls to be experimented, with each tested twice. The five varying radii balls used are: 0.037m, 0.052m, 0.063m, 0.074m, 0.080m. The four varied masses used are: 0.0200kg, 0.0400kg, 0.0600kg, 0.0800kg.

Materials:

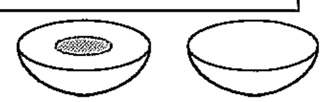
- vacuum cleaner with reverse cycle
- polystyrene balls of radius: 0.0374m, 0.0517m, 0.0629m, 0.0740m, 0.0796m (4 of each)
- hot wire
- two large clamps
- sturdy bench
- ≈ 10-20cm wood (4cm thick)
- Motion Detector (with Macmotion computer program) and computer
- ≈ 1kg of lead pellets
- disposable gloves
- sharp toothpicks
- PVC glue
- scalpel
- spatula
- electronic scales

Method:

A – Preparing different massed balls

Although balls of different sizes but same texture were readily available, balls of different masses were not. The balls had to be manually prepared to their required masses. The following a suitable method, given available materials.

1. Using the hot wire, cut all foam balls in half.
2. Using scalpel, scrape out a portion of the inside of one half of each ball

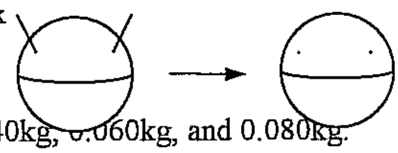


3. On a scale (with 5×10^{-5} *kg* error), place both halves of the balls and two toothpicks, then add glue to one half. Therefore all masses are taken into consideration.

4. Wearing gloves (for safety), use a spatula to add lead pellets to the newly made hole, until the scale reads 0.0200kg.

5. Push the two halves of the ball together

6. Push toothpicks right in (so the ball's surface is not affected) so there is extra stability.



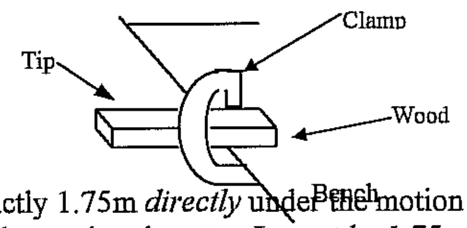
7. Repeat steps 3 – 6 for each ball size.

8. Repeat steps 3 – 7, except filling with lead to masses of 0.040kg, 0.060kg, and 0.080kg.

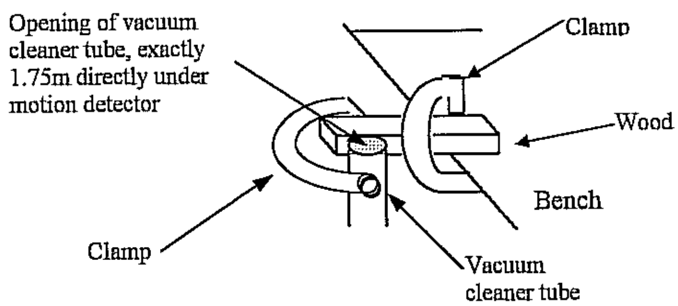
B – Setup of equipment

9. Stick the motion detector onto the roof with velcro, and connect all parts of the motion detector using booklet instructions. Set up the computer to *Macmotion.*

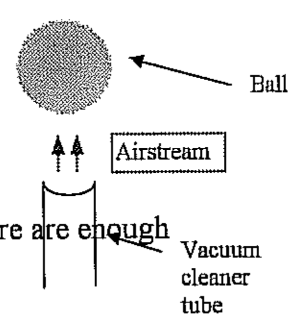
10. Clamp a piece of a wood onto a bench, the tip ≈ 1.75m vertically under the motion detector.



11. Clamp the tube of the vacuum cleaner onto the wood exactly 1.75m *directly* under the motion detector to ensure that the ball can later be picked up by the motion detector. It must be 1.75m under for later calculations.



C – Experimentation



12. Put vacuum cleaner on reverse cycle so it blows out air

13. Place ball in path of airstream.

14. Click "start" on computer and let the graph run for 10 seconds. Thus, there are enough oscillations to obtain accurate results. Save file when graph completed.

Repeat steps 12-14 twice each to ensure accuracy in original measurement, using each of the 20 balls.

Results*:

The following shows: average height of ball, amplitude (zero-peak) of oscillation and period of oscillation.

Ball (± 5 x 10^{-4}) *(m)*	*mass* (± 5 x 10^{-6}) *(kg)*	*average height* (± 0.02) *(m)*	*Amplitude* (± 0.02) *(m)*	*Period* (± 0.05) *(s)*
1	0.020	0.340	0.065	1.1
r = 0.037	0.040	0.120	0.045	1.35
	0.060	0.053	0.0125	2.1
	0.080	0.015	-	
2	0.020	0.462	0.065	1.3
r = 0.052	0.040	0.222	0.115	1.2
	0.060	0.062	0.045	1.075
	0.080	0.027	-	
3	0.020	0.604	0.0425	1.31
r = 0.063	0.040	0.339	0.055	1.25
	0.060	0.127	0.0625	1.38
	0.080	0.064	-	
4	0.020	0.687	0.06	1.58
r = 0.074	0.040	0.372	0.07	1.36
	0.060	0.225	0.0975	1.4
	0.080	0.062	0.025	1.13
5	0.020	0.771	0.06	1.64
r = 0.080	0.040	0.476	0.095	1.42
	0.060	0.266	0.075	1.34
	0.080	0.098	0.0625	1.37

How errors were obtained:
mass: error of electronic scale
amplitude: furthest amplitude from mean
height: determined from amplitude error
period: furthest period value from the mean period value.

** Raw data (original graphs) in appendix 1*

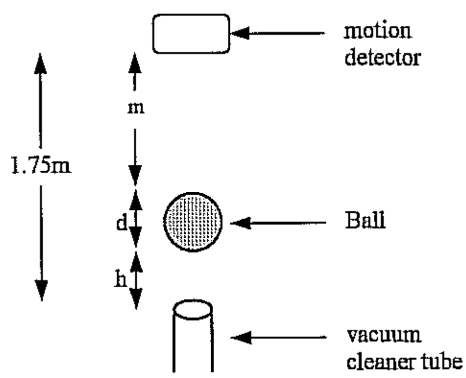
Calculations & Discussions:

HEIGHT:

It was originally thought that height would be constant, however it oscillated, producing fairly smooth sinusoidal curves. So this was investigated further (see "amplitude" and "period").
The average height of each ball was calculated by firstly manually drawing in average peak lines, eg.

Ball 5 (radius 0.080m), 0.080kg:

Using a spreadsheet, these peak values were added then divided by two, giving the ball's average distance from the motion detector (say, m) as shown:



Thus, to obtain the actual height from the tube (h), the following calculation was performed:

$$h = 1.75 - m - d \qquad \text{(done on a spreadsheet)}$$

It was found that the bigger balls gave smoother, more regular oscillations, eg:

Ball 5 (radius 0.080m), 0.060kg

In this particular ball's case, the starting few peaks have a slightly larger amplitude, because during the first few seconds the ball was stablising itself from the introduction of the force. This does not greatly affect results because although amplitude is slightly different, the average height remains fairly constant.

Smaller balls were not entirely be picked up by the motion detector. Instead, the 1.75m distance of the tube opening was picked up quite frequently, and the floor to a lesser degree, eg.

Ball 1 (radius 0.037m), 0.040kg

In these cases, the sinusoidal oscillations were pencilled in:

then normal calculations performed.

In some cases, however, it appeared that no oscillations occurred, so a single straight line was drawn at the smallest distances picked up , eg:

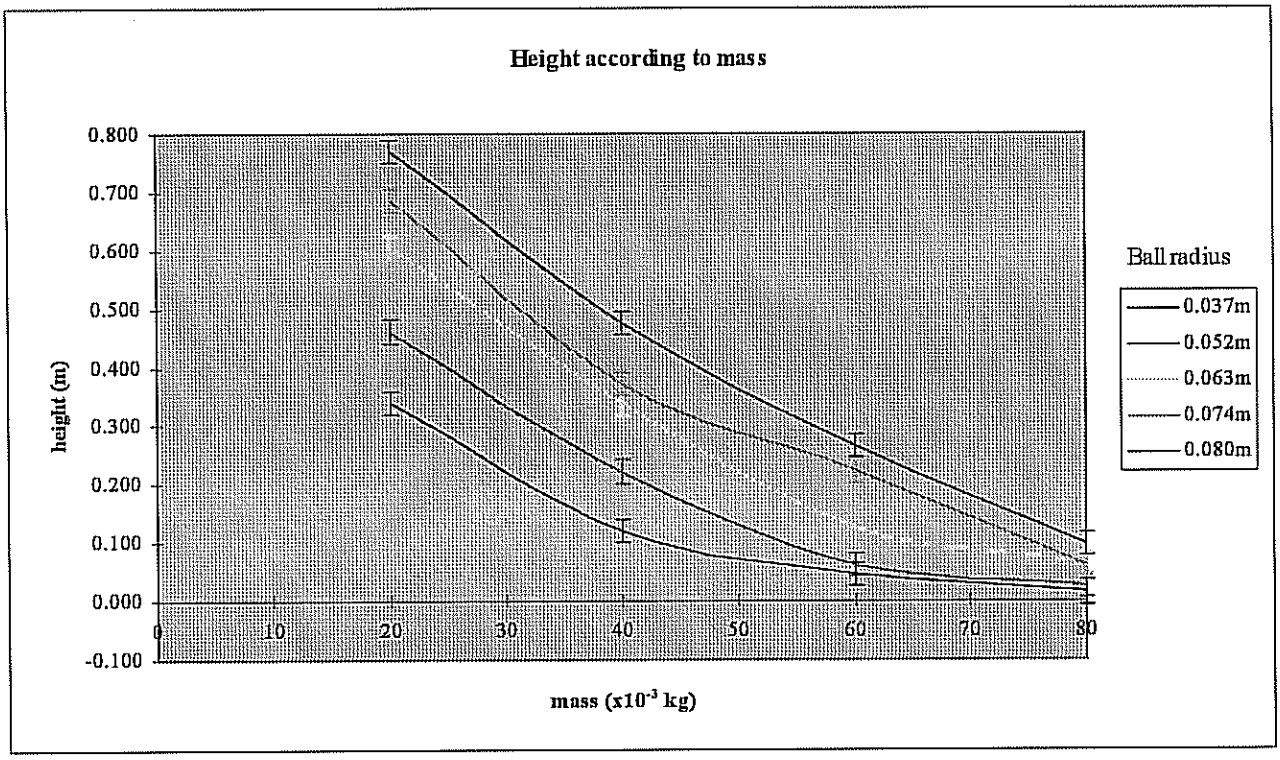
Ball 1 (radius 0.037m), 0.080kg

and this taken as the average height. This seemed suitable since the 1.75m distance the motion detector frequently picked up would have been the vacuum cleaner tube. The ball was the closest object to the motion detector, and therefore the smallest distances picked up on the graph. Despite the motion detector's inability to pick up balls at times, it is a very accurate tool.

With all these considerations, graphs of height according to mass and radius were drawn.

Mass:

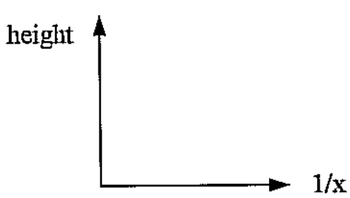
The following graph shows the average height of the ball according to mass:



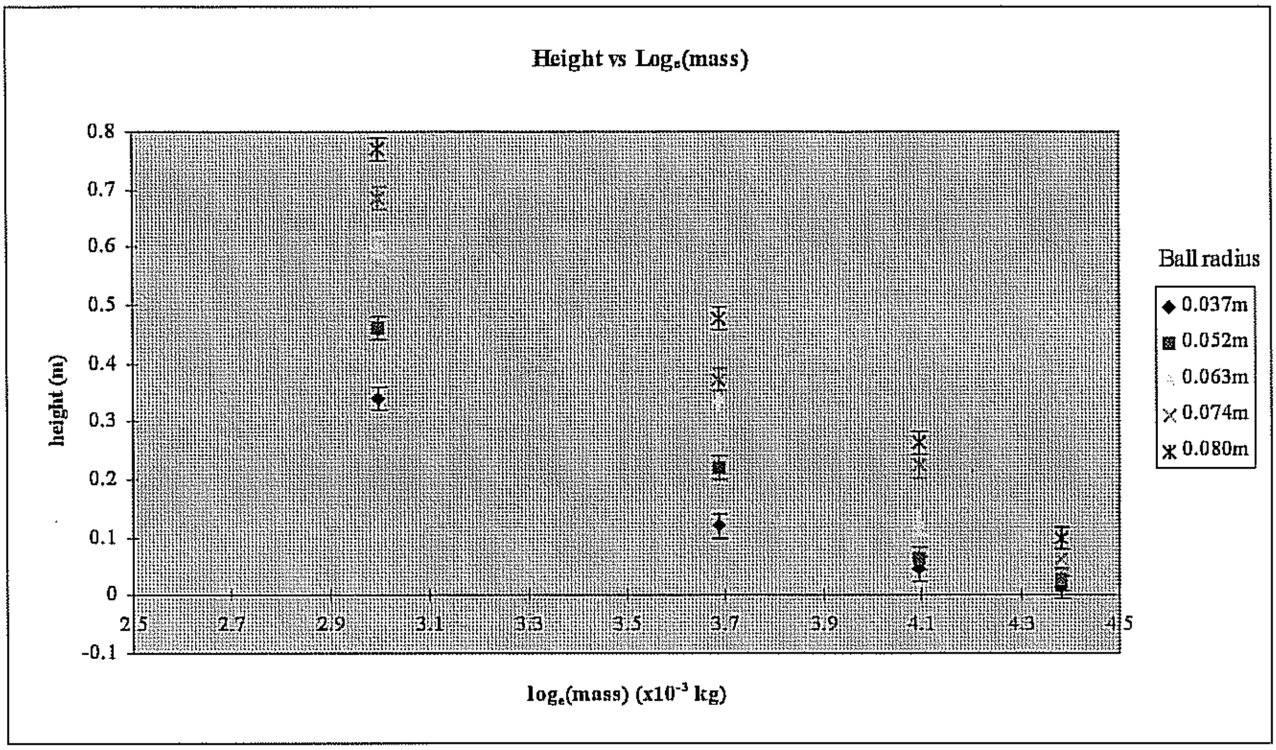
(See appendix for spreadsheet used to draw graph)

This graph shows a general decrease in height as the mass of the ball increases. This is because the downwards gravitational force is greater for balls with greater masses (force = mass x acceleration due to gravity), so the upwards force of the air cannot push them as high as lighter balls.

To find a relationship, graphs of *height vs. function of mass* were plotted, eg.

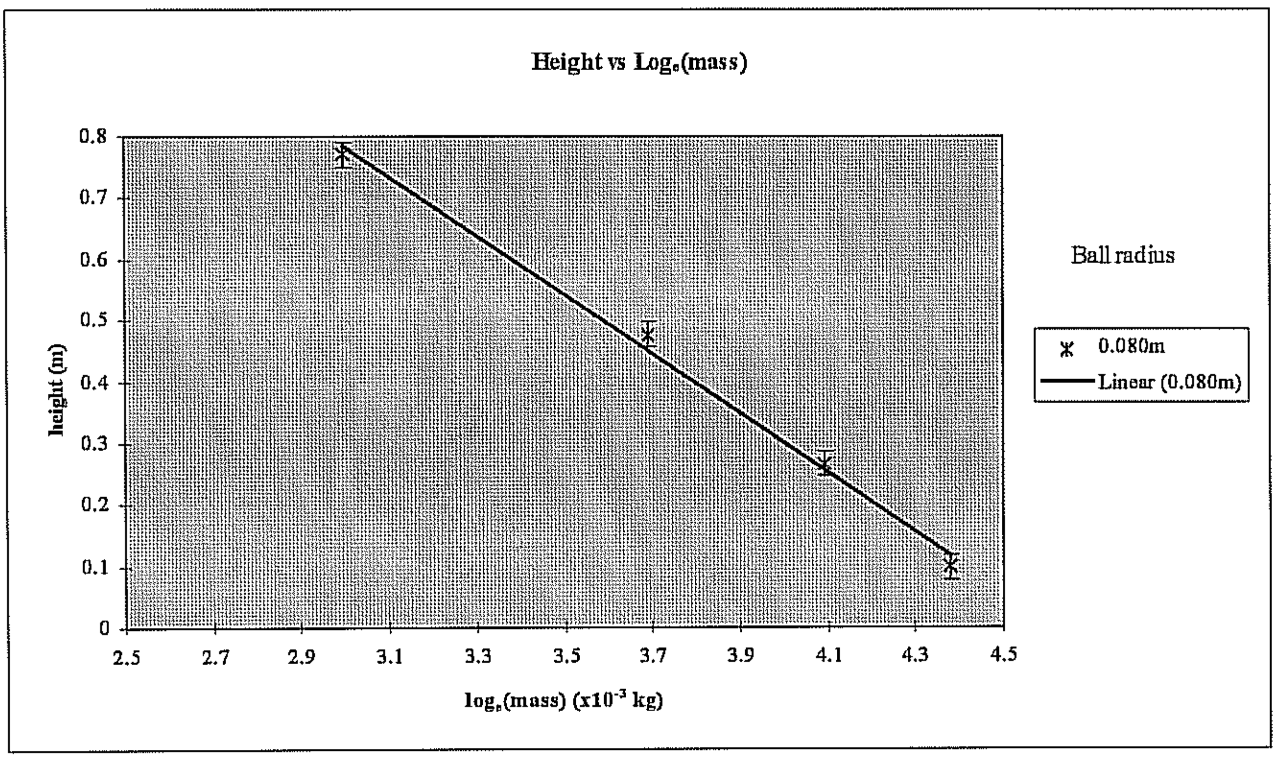


and the following linear graph was obtained after trying $\frac{1}{\text{mass}}$, $\frac{1}{\text{mass}^2}$, e^x and $\log_e x$:



It can therefore be said that function of height according to mass is logarithmic.

Taking out the specific graph of height vs log$_e$(mass) of radius 0.037m, and drawing a line of best fit:



It is known that for a linear equation: $y = mx + c$ (m=gradient, c = y-intercept)

$\therefore$ height = $m\log_e(mass)$+c

$m = \frac{rise}{run} = \frac{0.73-0.25}{3.1-4.1} = \frac{0.48}{-1.0} = -0.48$

height = $m\log_e(mass)$ + c

(4.1, 0.25) $\therefore$ 0.25 = - 0.48(4.1) + c $\Rightarrow$ c = 2.218 = 2.2 (2 significant figures)
$\therefore$ For a ball with radius 0.37m, height = -0.48$\log_e$*(mass)* + 2.2

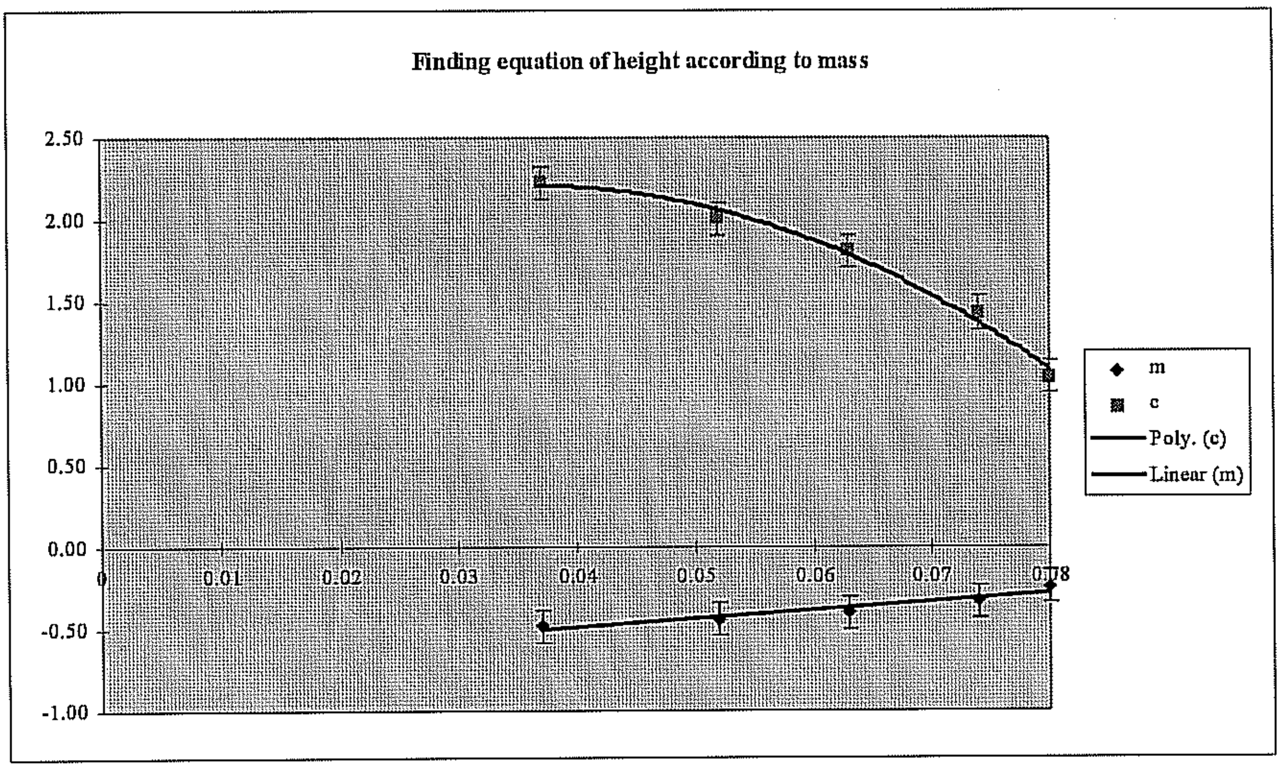
Similarly,

radius 0.052m: *height* = -0.44 $\log_e$*(mass)* +2.01
radius 0.063m: *height* = -0.40 $\log_e$*(mass)* +1.81
radius 0.074m: *height* = -0.33 $\log_e$*(mass)* +1.43
radius 0.080m: *height* = -0.24 $\log_e$*(mass)* +1.04

A spreadsheet of *m* (gradient of function) and *c* (y-intercept of function) according to radius:

radius (m)	*m*	*c*
0.037	-0.48	2.23
0.052	-0.44	2.01
0.063	-0.40	1.81
0.074	-0.33	1.43
0.080	-0.24	1.04

giving the following graph:

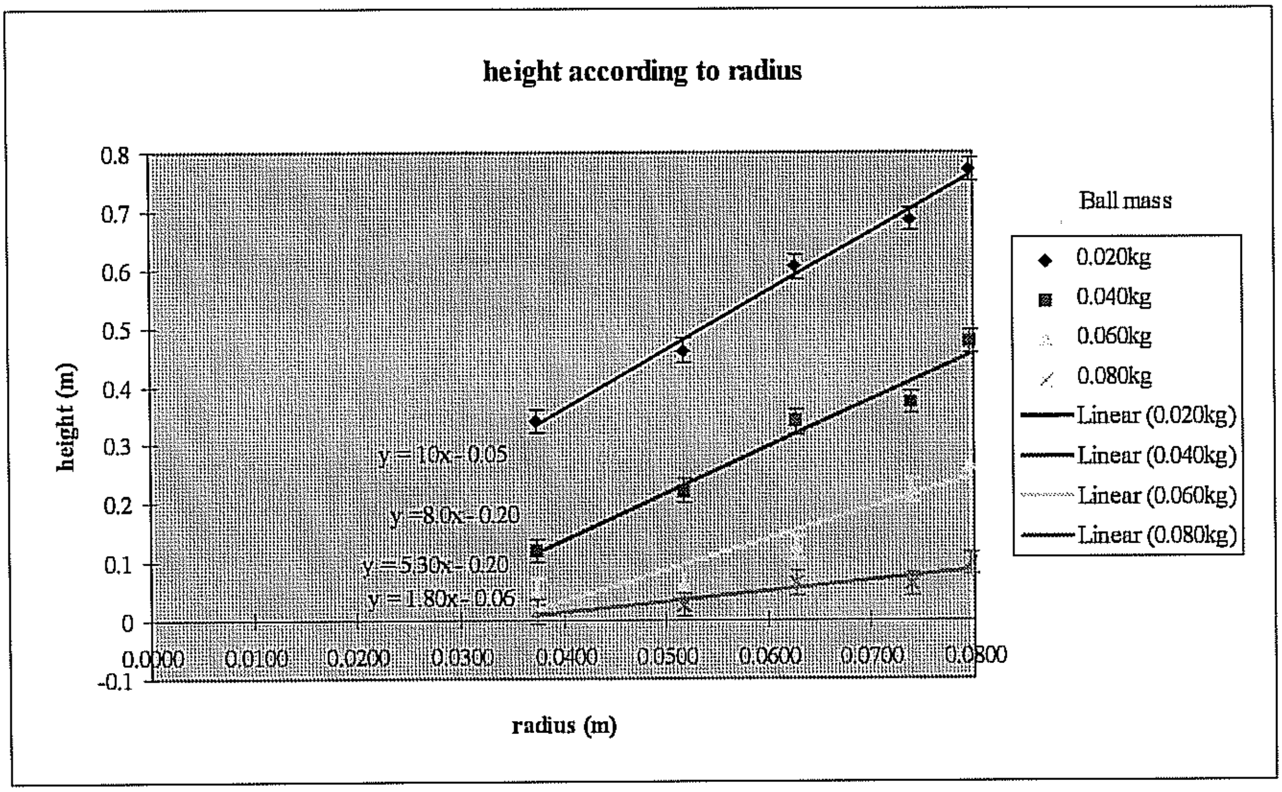


(see appendix to see how equations were derived)

It can therefore be deduced that:
***height* = (5.2r – 0.70) $\log_e$(*mass*) + (-2.6r + 3.3)**
This is the general equation (approximated to 2 significant figures) for height given mass.

Radius

Points of height according to radius were plotted. The following graph shows linear trendlines through the points which were found in the experiment. It shows how height varies with radius of the ball.



(See appendix for spreadsheet used to draw graph)

The linear trendlines fail to go through a few error bars here. This would mainly be due to inaccurate measurements, the cause of draughts from openings of doors, and people walking by, as this would have affected height measurements.

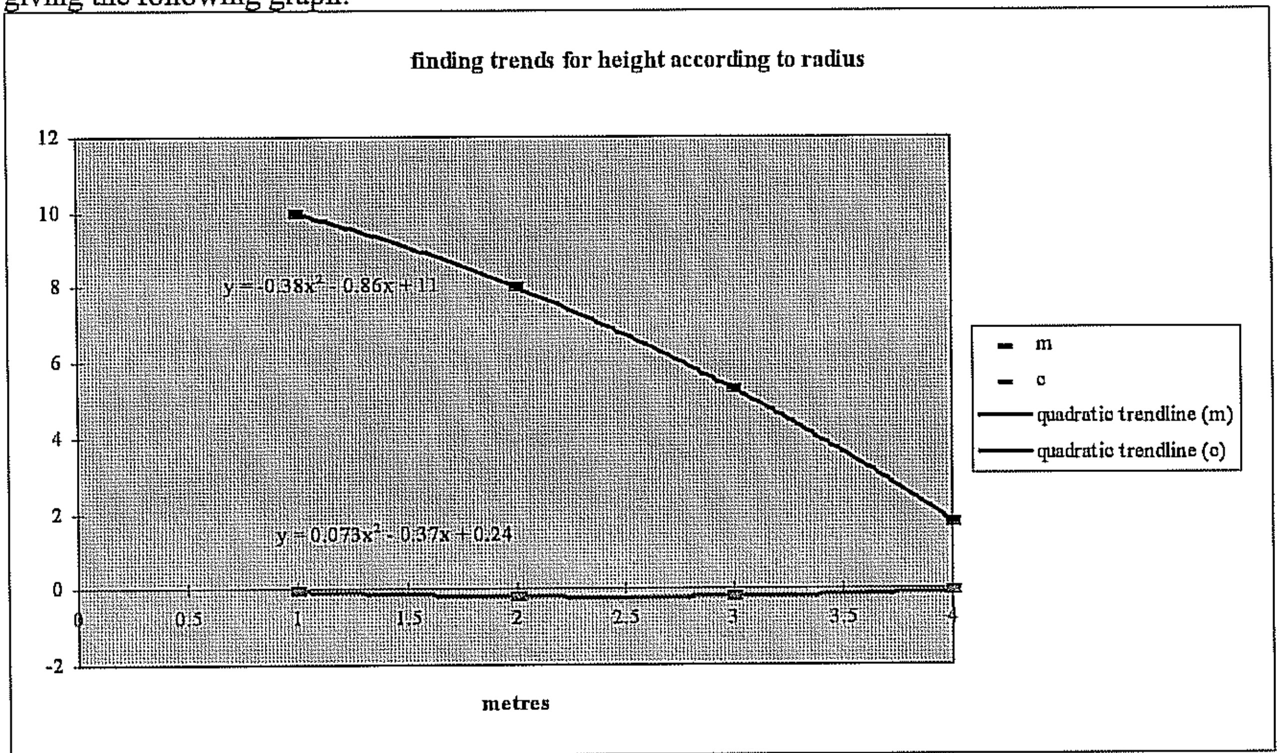
Equations were found in a similar manner to that of finding mass in relation to height.

Summarising the equations
0.020kg ball: height = 10r – 0.05 (r = radius)
0.040kg ball: height = 8.0r – 0.20
0.060kg ball: height = 5.30r – 0.20
0.080kg ball: height = 1.80r – 0.06

A spreadsheet of *m* (gradient of function) and *c* (y-intercept of function) according to radius:

ball	*m*	*c*
0.020kg	10	-0.050
0.040kg	8.0	-0.20
0.060kg	5.3	-0.20
0.080kg	1.8	-0.060

giving the following graph:



(see appendix to see how equations were derived)

It was calculated that height errors bars for this graph were 0.1.

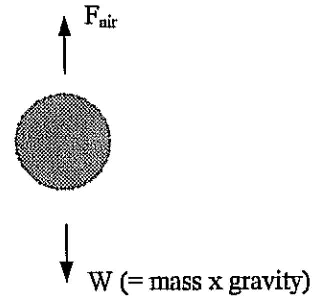
It can therefore be deduced that:

$$height = (-0.38r^2 - 0.86r + 11) \log_e(mass) + (0.073r^2 - 0.37r + 0.24)$$

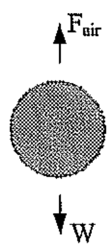
This is the general equation (approximated to 2 significant figures) for height given radius.

AMPLITUDE

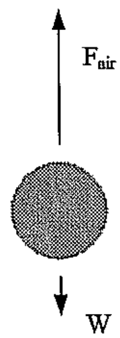
The following vertical forces act on the ball:



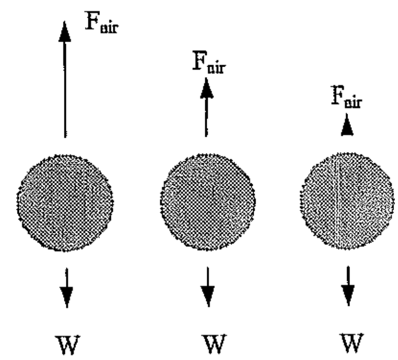
If $F_{air} = W$, forces are balanced and balls would remain at constant heights.



However, due to the intial upwards force from the vacuum cleaner, the ball had unbalanced forces acting on it, so moved upwards, $\therefore F_{air} > W$

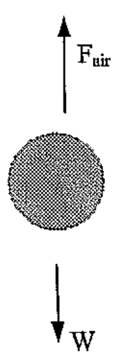


As the ball moves up, it also moves further from the source of F_{air}, $\therefore F_{air}$ decreases, until $F_{air} < W$



The ball is then close to F_{air} again, $\therefore F_{air} > W$
This cycle is continually repeated, causing the ball's vertical oscillation.

When mass increases, W increases:



Therefore, the initial F_{air} on the ball does not cause as great a lift, as the downward force of W is greater for heavier balls. As the ball moves upwards, F_{air} decreases, but not as significantly. Since F_{air} has not greatly decreased, the ball does not go down as far. Therefore, oscillation is over a much smaller range, and balls with greater masses should have smaller amplitudes.

Amplitude, in this experiment, was found by the manually drawn in average peak lines (done in finding average height), then performing the calculation:

$$\text{Amplitude} = \frac{\text{Peak} - \text{Trough}}{2}$$

eg,

Ball 5 (radius 0.080m), 0.080kg:

$$Amplitude = \frac{Peak - Trough}{2}$$

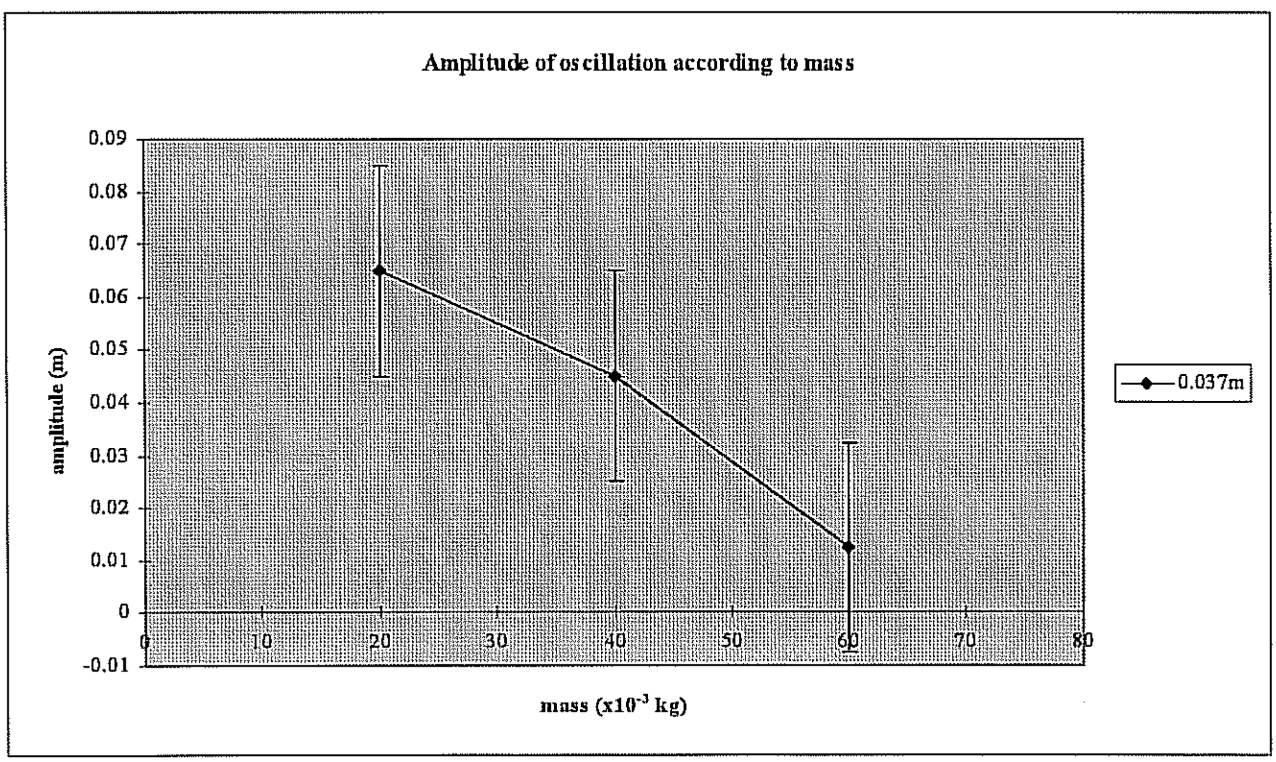
$$Amplitude = \frac{1.55 - 1.42}{2}$$

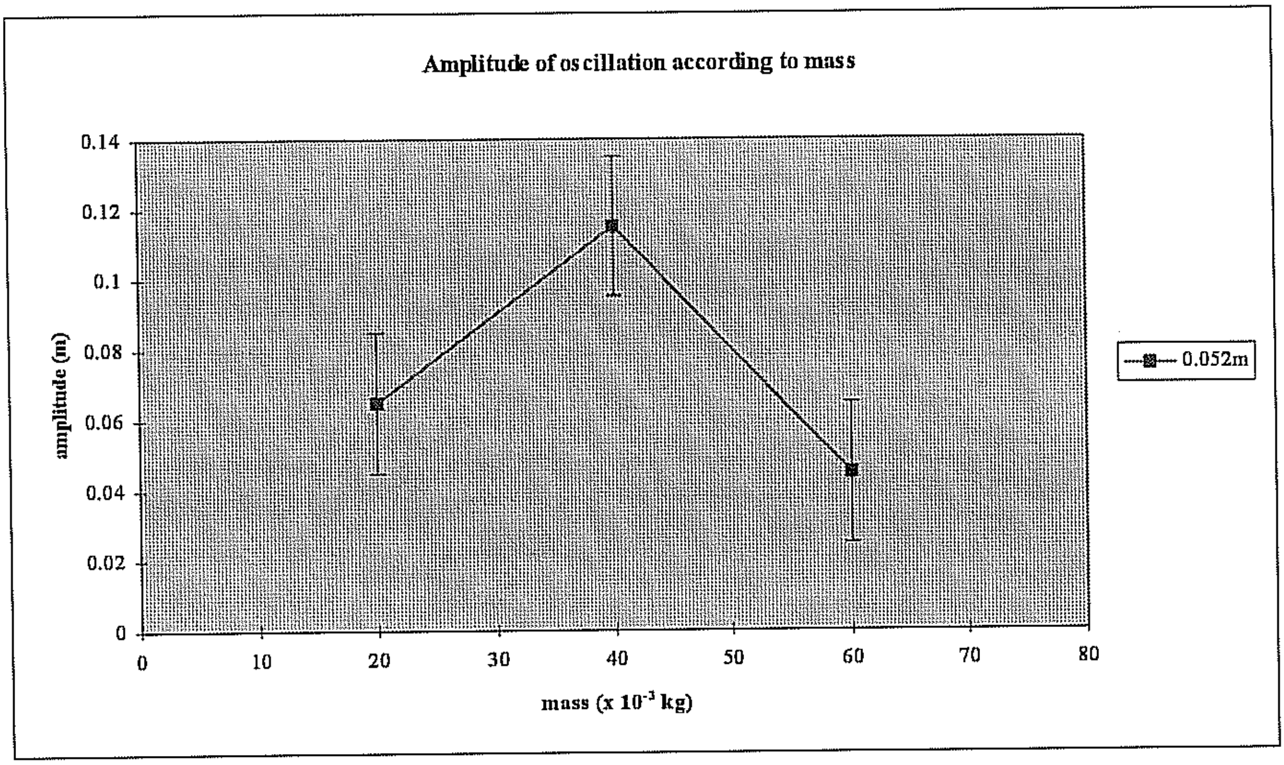
$$Amplitude = 0.065\ (m)$$

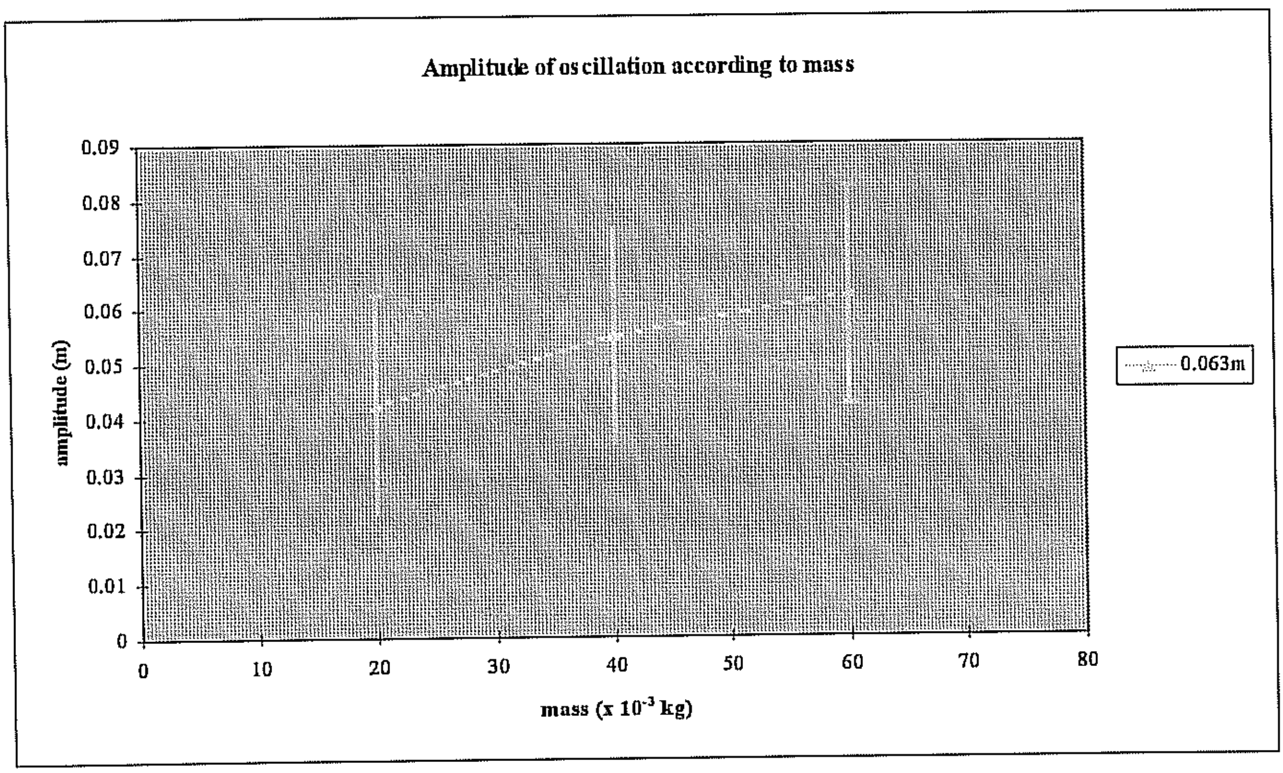
Graphs of amplitude according to mass and radius were drawn using spreadsheets.

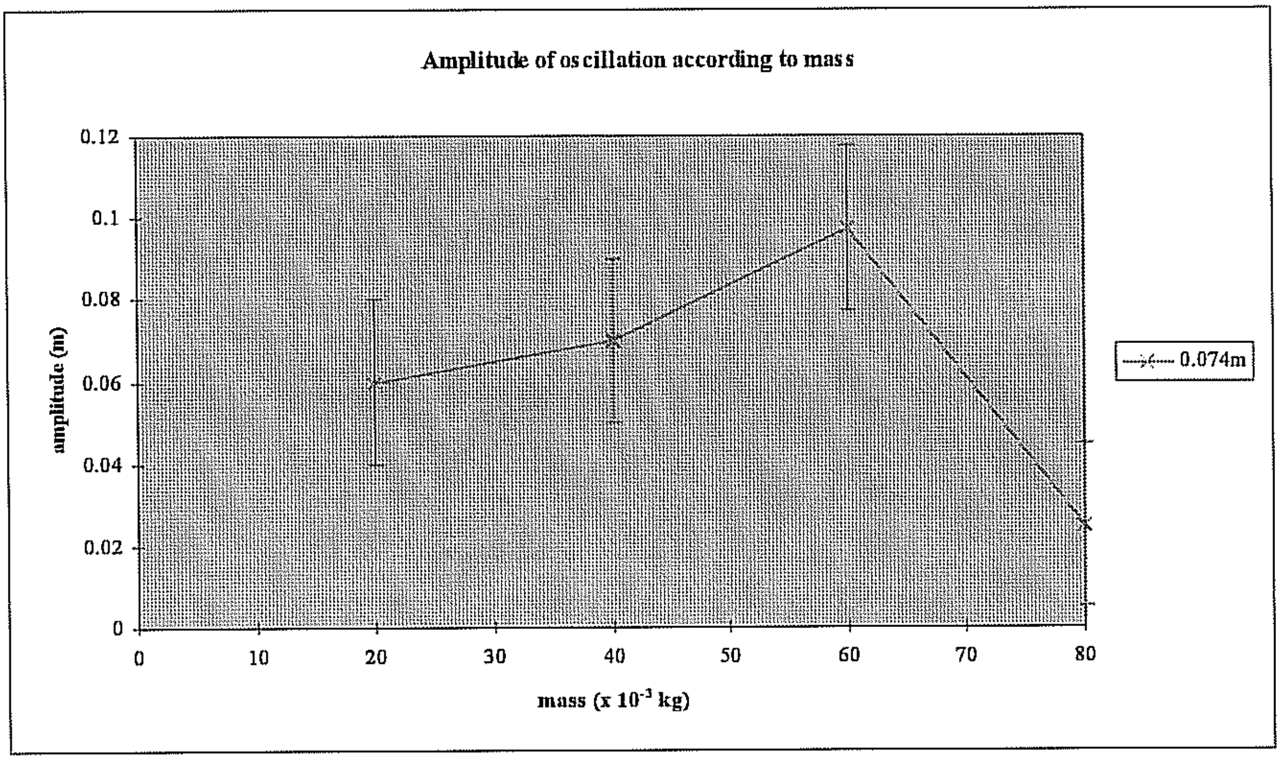
Mass:

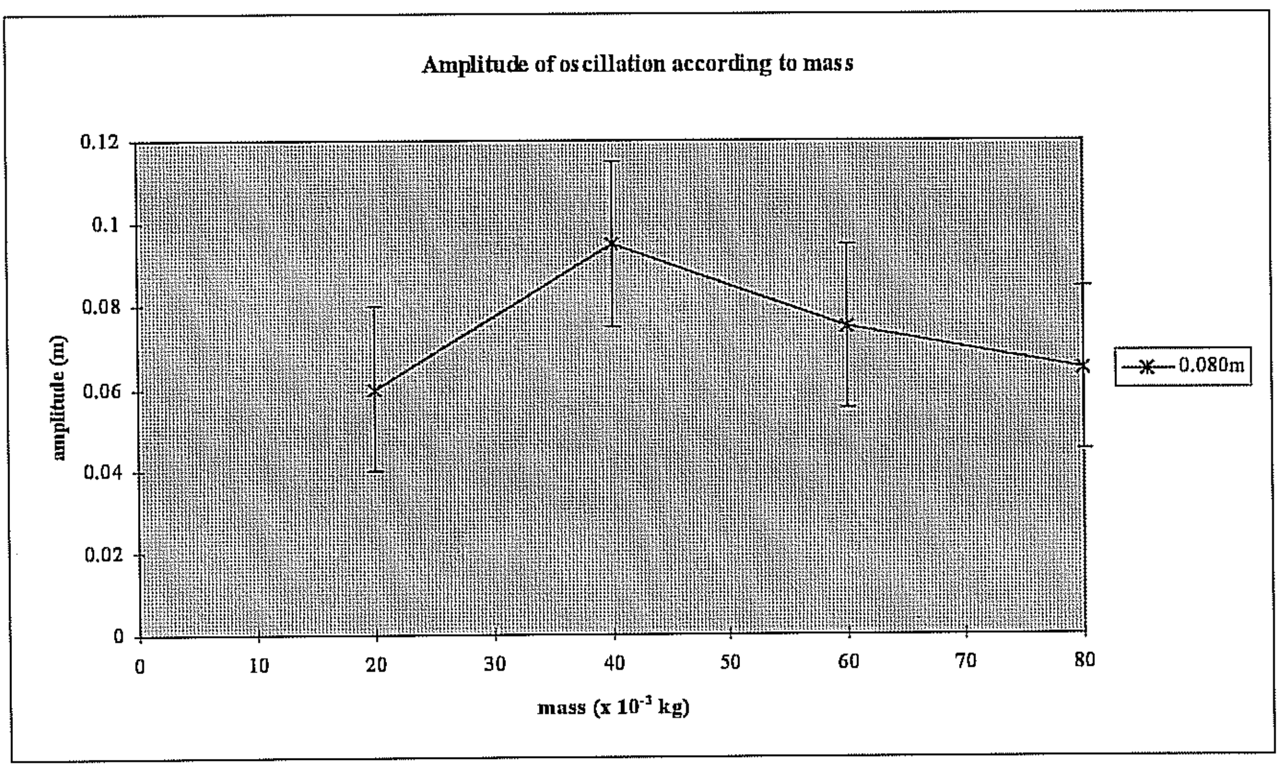
The following graphs show amplitude according to ball mass for each ball radius.




Amplitude of oscillation according to mass
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
amplitude (m)
0
10
20
30
40
50
60
70
80
mass (x 10^-3 kg)
0.052m


Amplitude of oscillation according to mass
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
amplitude (m)
0
10
20
30
40
50
60
70
80
mass (x 10^-3 kg)
0.063m

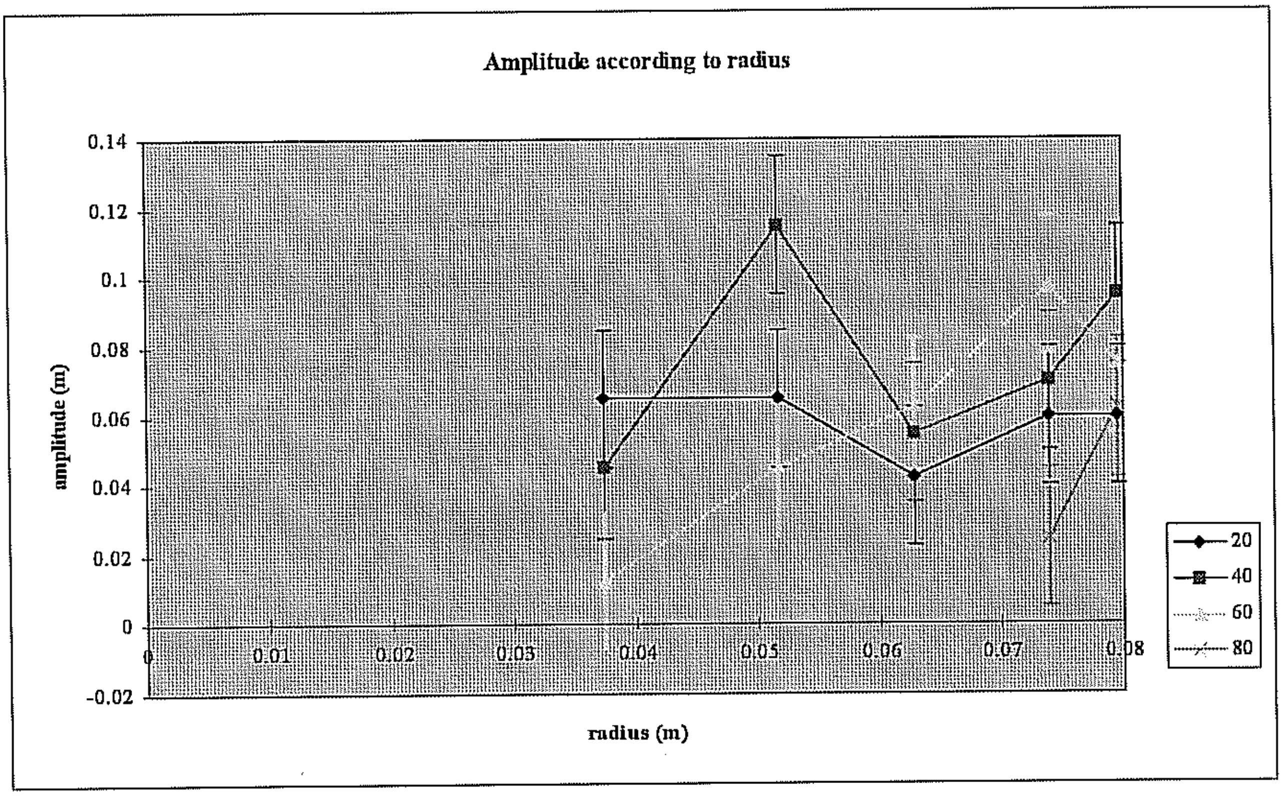




Within the error bar regions, some of these graphs show positive linear relationships, others negative, or even quadratic. Predictions of decreased amplitude as mass increased were not found for all sized balls. This is probably due to inaccurate measurements due to draughts from opening doors and people passing by, and without this perhaps a relationship would have been found.

Therefore no relationship can be found between amplitude and mass.

Mass:



Again, there appears to be no distinct relationship within the error bar region.

Therefore a relationship between amplitude and radius was not found.

PERIOD

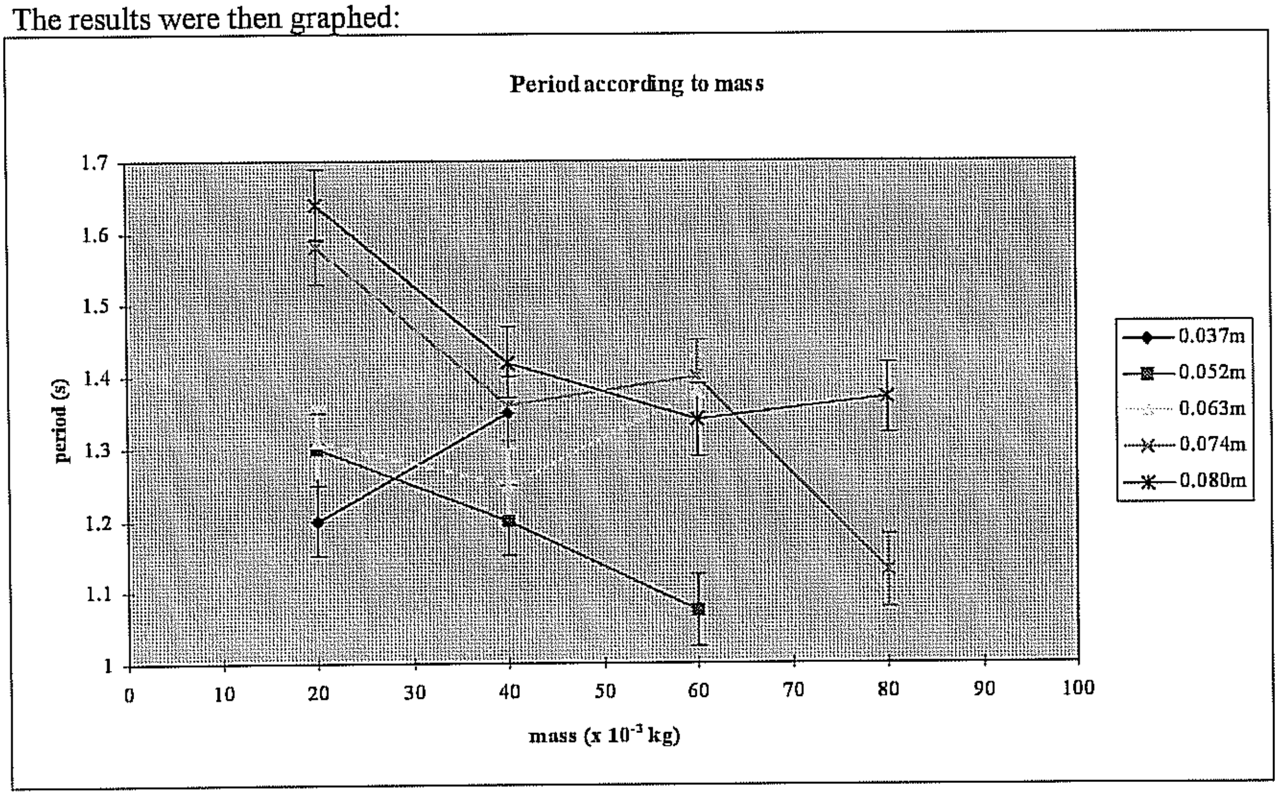
The period is how long it took for one oscillation, found by drawing horizontal lines through two troughs, and dividing the time interval between them by the number of oscillations, eg.

Ball 3 (radius 0.063m), 0.060kg:

$$\therefore \text{Period} = \frac{\text{trough}_2 - \text{trough}_1}{\text{time interval between troughs}}$$

$$=$$

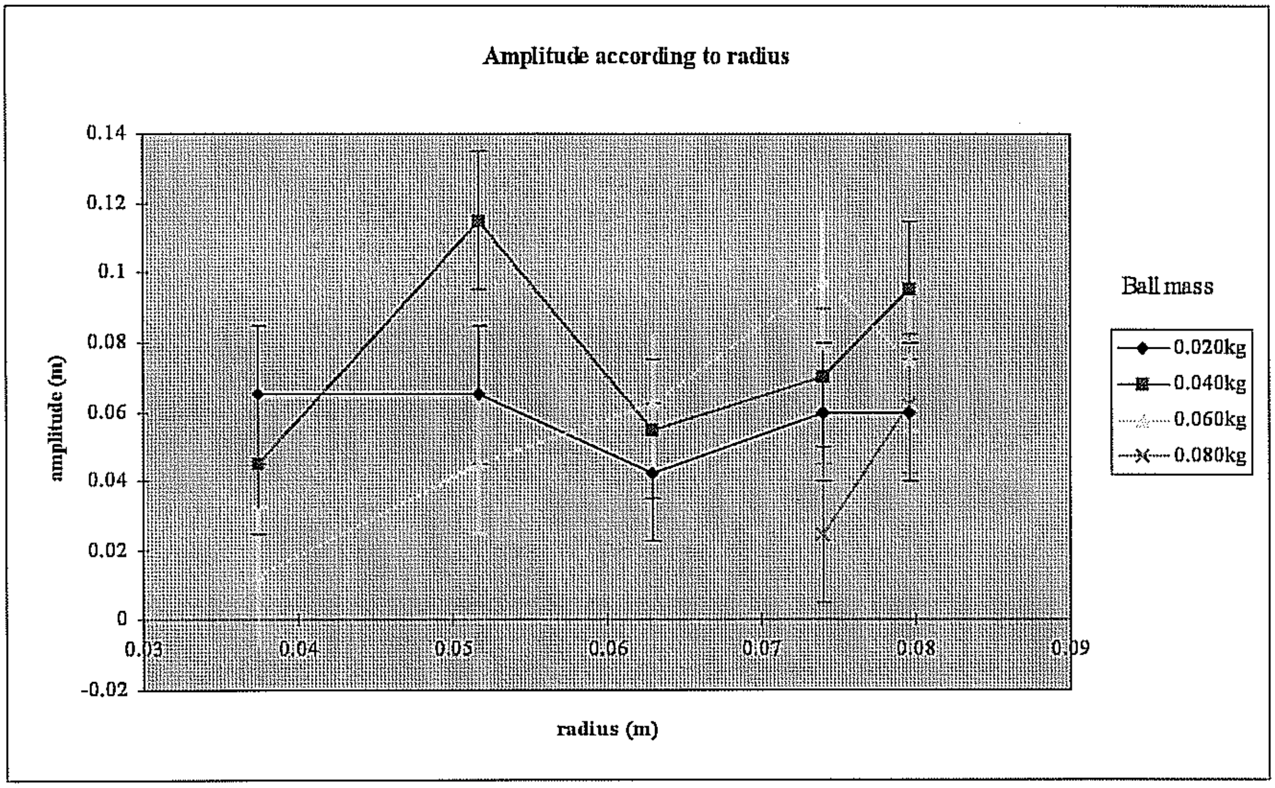
The results were then graphed:



Althought there appears to be a somewhat linear relationship here, the gradients are all different and the data too scarce to sufficiently ascertain a relationship between period and mass.

Scarcity of data was due to the motion detector incompletely detecting smaller sized balls, so no sinusoidal graphs drawn for these balls (hence no period). Furthermore, data may have been affected by draughts which were inevitable due to classroom activity. Had the balls been picked up by the motion detector, or experimentation performed in an enclosed room, a correlation may have been found.

A graph of period according to radius:



There looks like a positive linear relationship. However, again there is not sufficient data to make a proper correlation.

Limitations:

Factors which may have affected experimental results but not taken into consideration:

- Draughts from opening of classroom door or people walking by may have affected the velocity of the air coming out of the vacuum cleaner, or the amplitude or period of oscillation

Factors which may have caused an effect on the experiment to a much lesser extent:

- Atmospheric pressure changes (affect density of air)
- Temperature changes (affect density of air)
- Power of the vacuum cleaner due to variation in AC current

These factors were tried to be kept as constant as possible by experimenting each set of experiments on the one day, and repeated experiments performed all on another day. Since the results for each appeared to be the same, it was concluded that these outside variables did not have a significant effect within the number of significant figures used.

Suggestions for further investigation:

Further extensions could include varying factors such velocity and force of airstream, using balls of different textured surfaces, varying sizes of the vacuum tube or putting the tube at an angle.

Improvements to the investigation include:

- not handmaking the massed balls (to save time and improve accuracy)
- experimenting with bigger balls – this would provide both graphs with smoother sinusoidal curves, and clearer graphs, as bigger balls get picked up by the motion detector much better
- experimenting within an enclosed room with constant temperature and pressure and no draughts.

Conclusion:

Relationship between height and mass:

$$\textit{height} = (5.2r - 0.70)\log_e(\textit{mass}) + (-2.6r + 3.3)$$

Relationship between height and radius:

$$\textit{height} = (-0.38r^2 - 0.90 + 11)\log_e(\textit{mass}) + (0.07x^2 - 0.37x + 0.24)$$

Relationship between amplitude and mass: *none found*

Relationship between amplitude and radius: *none found*

Relationship between period and mass: *none found*

Relationship between period and radius: *none found*

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